Utilizing Gauss-Legendre Quadrature for Computation of Radiative Fluxes in Atmospheric Models

Howard W. Barker   Jiangnan Li   Jason Cole

Environment and Climate Change Canada   Environnement et Changement Climatique Canada

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- GCMs RT models (Li and Barker 2018):

  - sort stochastic CWP... low-order GLQ... SW and LW operate on common atmos.
  - reduce noise for NET fluxes... boundary fluxes OK; HRs not so much

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- GCMs RT models (Li and Barker 2018):

  - sort stochastic CWP... low-order GLQ... SW and LW operate on common atmos.
  - reduce noise for NET fluxes... boundary fluxes OK; HRs not so much

- State-of-the-art LESs, CSRMs, NWP models run routinely using:

  - **domain sizes...... 50 - 1,000 km**
  - **grid-spacings..... 0.1 - 1.0 km... 10^5 - 10^7 columns**
  - ***N* layers.......... 64 - 256**
  - **1D RT in ICA...... yes**
  - **RT timesteps...... every 15 - 30 dynamics timesteps**
  - **RT of total CPU... 15 - 35%**

- an attempt at a catastrophic reduction of CPU time consumed by RT algorithms

(per. comm. S. Krueger, J. Manners, P. Vaillancourt, 2018)

Dealing with RT's CPU demand

1. full ICA... but only every *N* dynamics timesteps (probably the most common?)
2. full ICA... as in (1) but intermediate steps using CRE bands only (Manners et al.)
3. ICA... employing stochastic spectral sampling (Pincus et al.)

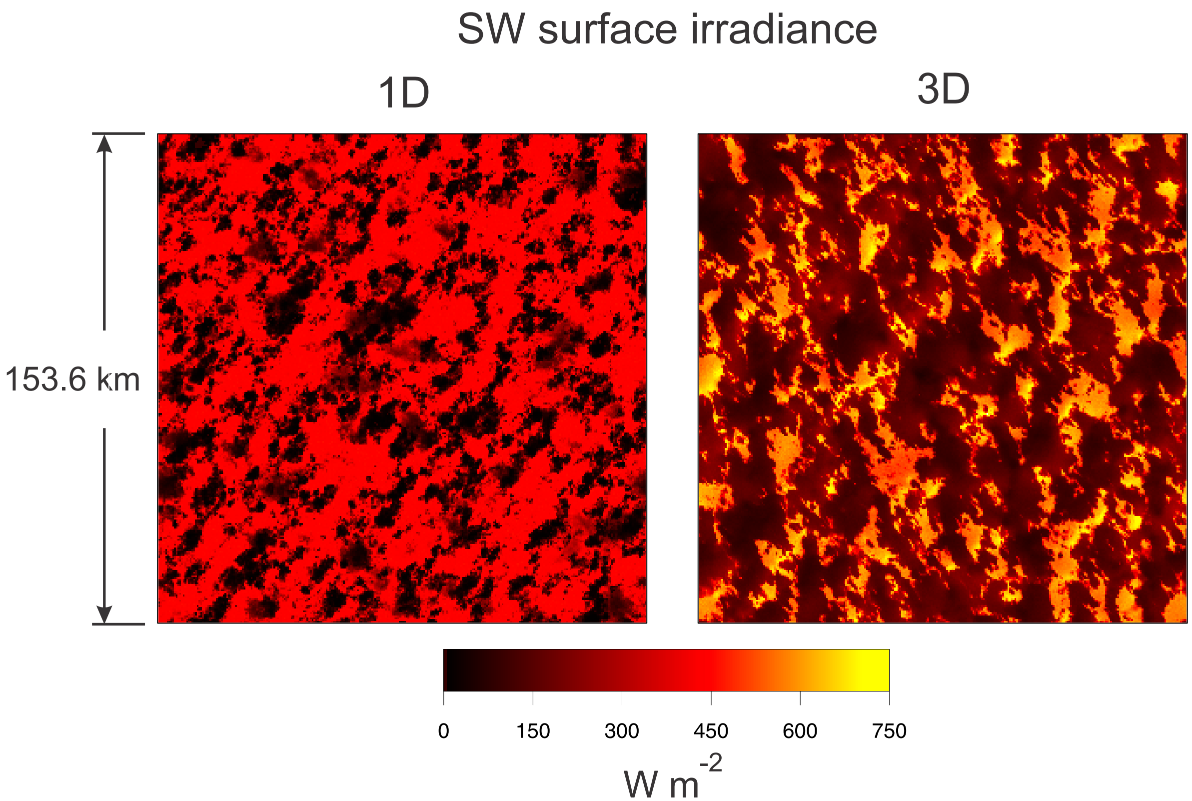
Goal:

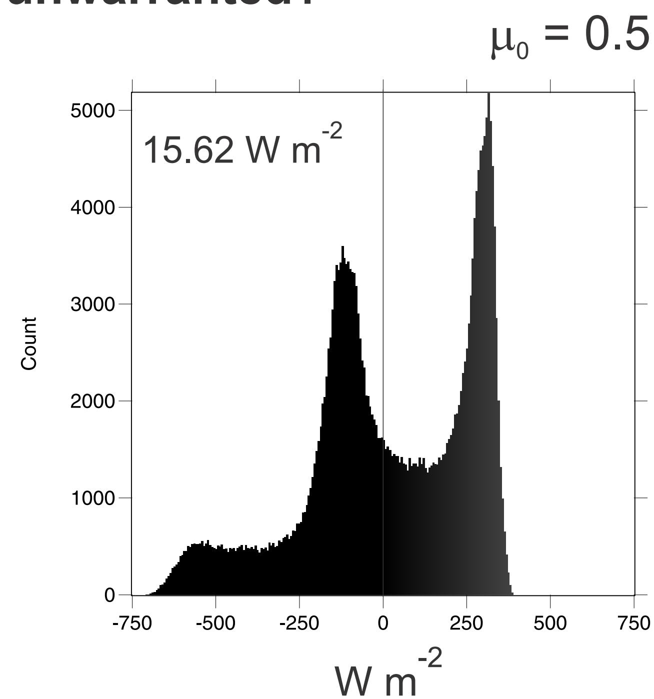
Full-resolution (time, space, spectra) Q_{rad} using much less CPU time than the above methods and resulting in simulated (cloud) properties that differ insignificantly from those obtained with the full-ICA.

- Towards the modelling "chasm" as described by Lawrence et al. (2018)
- NB. still adhering to the1D-ICA paradigm...

Something to bear in mind...

Is moving to 3D RT considered to be intractable or unwarranted?





Something to bear in mind...

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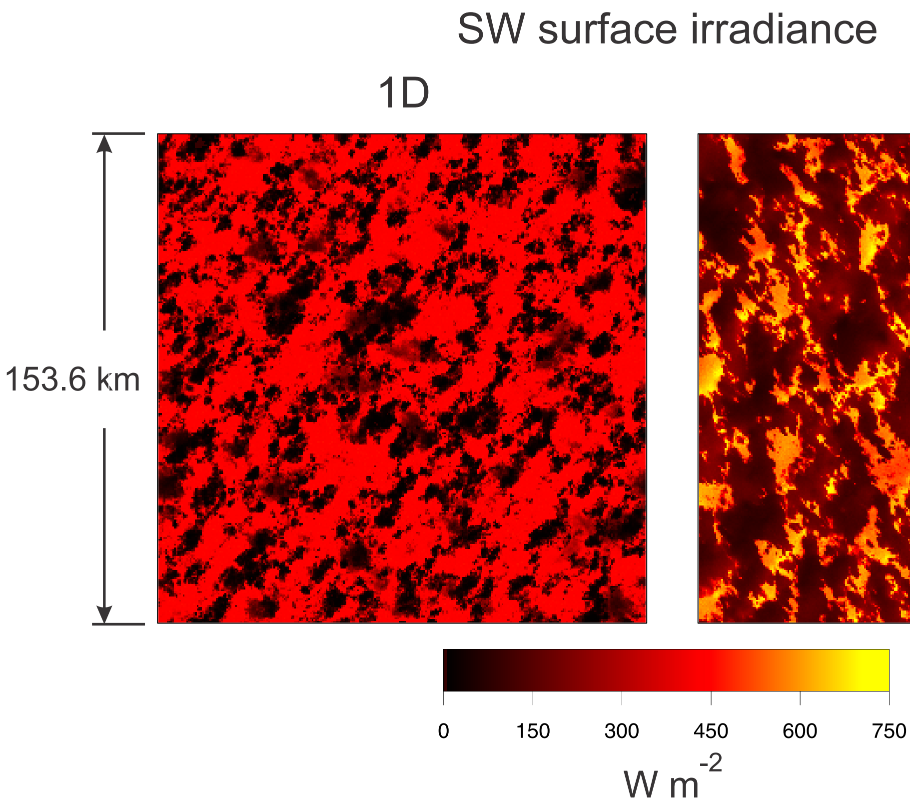
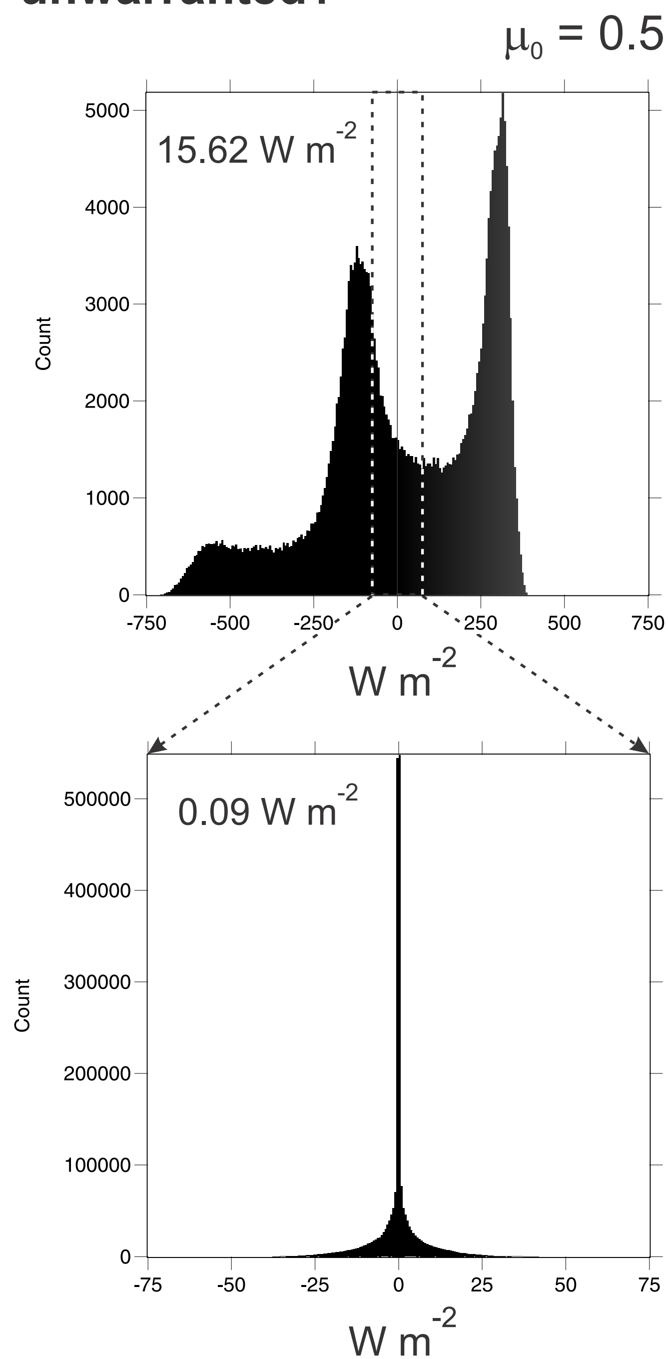




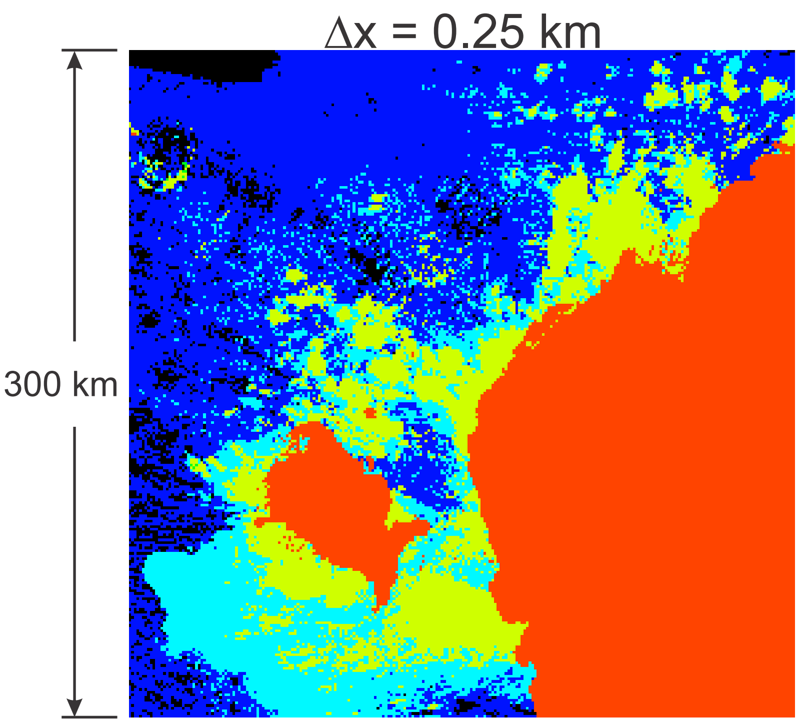
If yes, then the 1D RT errors to be shown have to be considered almost negligible!

1. Partitioning a domain's columns

Partition into sub-domains such that radiative flux profiles are "distinctive"

cf. *K*-means

** max(Q_{rad}) near cloudtops exposed-to-space



4.6% 0: cloudless

34.7% 1: cloudtops > 10 km

13.0% 2: 5 km < cloudtops < 10 km (liq + ice)

0.2% 3: 5 km < cloudtops < 10 km (ice-only)

14.9% 4: 5 km < cloudtops < 10 km (liq-only)

32.6% 5: cloudtops < 5 km

2. Sort columns within partitions

Partition into sub-domains such that radiative flux profiles are "distinctive"

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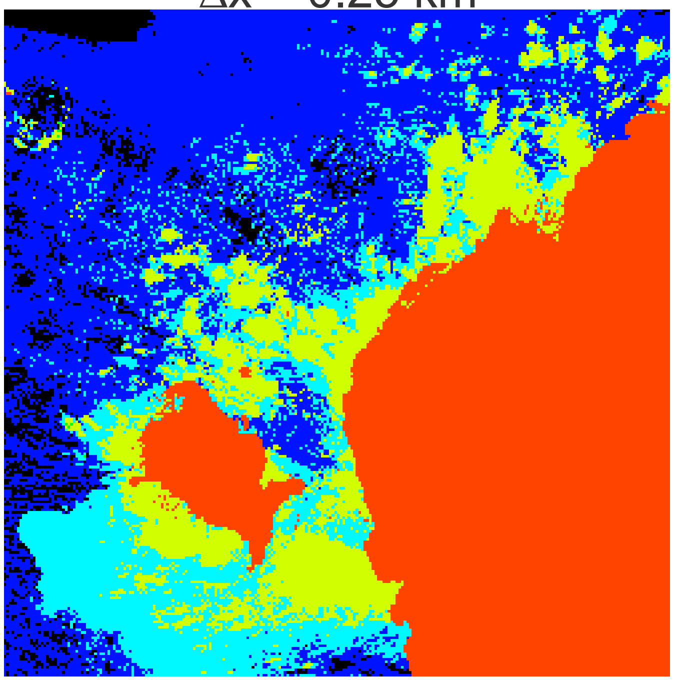
** max(Q_{rad}) near cloudtops exposed-to-space

Δx = 0.25 km

300 km



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32.6%		5: cloudtops < 5 km

$$\langle F\rangle = \frac{1}{\mathcal{N}}\sum_{n=1}^{\mathcal{N}} F(n)$$

← full ICA

$$= \frac{\sum_{m=1}^{\mathcal{M}} \mathcal{N}_s(m)\left[\frac{1}{\mathcal{N}_s(m)}\sum_{n=1}^{\mathcal{N}_s(m)} F_m(n)\right]}{\sum_{m=1}^{\mathcal{M}}\mathcal{N}_s(m)} = \frac{\sum_{m=1}^{\mathcal{M}}\mathcal{N}_s(m)\langle F_m\rangle}{\sum_{m=1}^{\mathcal{M}}\mathcal{N}_s(m)}$$

partitioned into $\mathcal{M}$ categories

for each partition, sort and index according to CWP

$$s_{m,n} = \frac{n-1}{\mathcal{N}_s(m)-1}; \quad n = 1,\ldots,\mathcal{N}_s(m)$$

$$\langle F_m\rangle = \int_0^1 F(s_m)\,\mathrm{d}s_m$$

3. Apply GLQ to sorted partitions

Partition into sub-domains such that radiative flux profiles are “distinctive”

cf. *K*-means

** max(Q_{rad}) near cloudtops exposed-to-space

$$\langle F \rangle = \frac{1}{\mathcal{N}} \sum_{n=1}^{\mathcal{N}} F(n) \quad \longleftarrow \text{ full ICA}$$

$$= \frac{\sum_{m=1}^{\mathcal{M}} \mathcal{N}_s(m) \left[\frac{1}{\mathcal{N}_s(m)} \sum_{n=1}^{\mathcal{N}_s(m)} F_m(n) \right]}{\sum_{m=1}^{\mathcal{M}} \mathcal{N}_s(m)} = \frac{\sum_{m=1}^{\mathcal{M}} \mathcal{N}_s(m) \langle F_m \rangle}{\sum_{m=1}^{\mathcal{M}} \mathcal{N}_s(m)}$$

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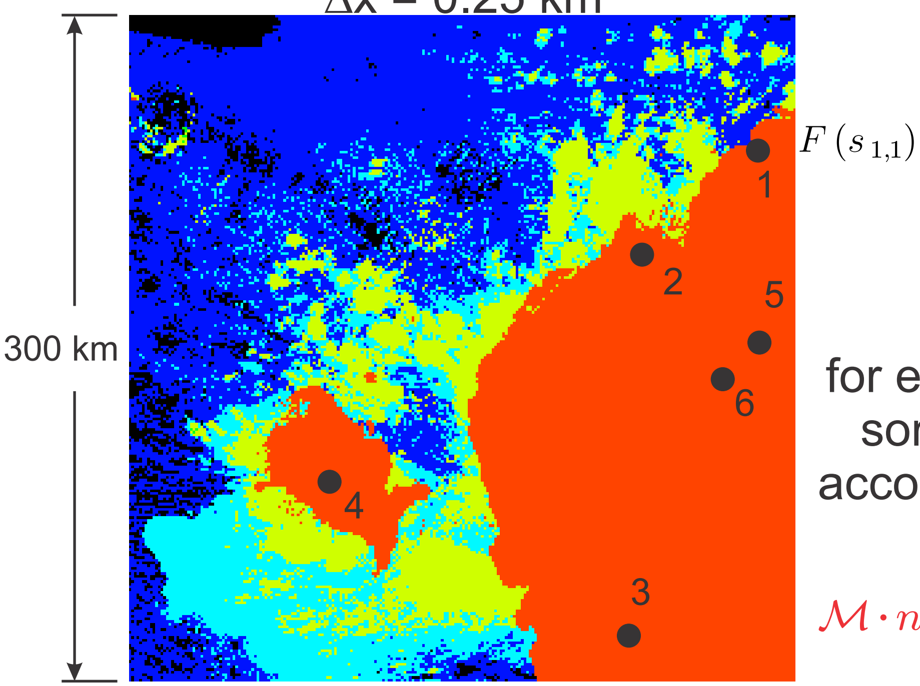

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$$s_{m,n} = \frac{n-1}{\mathcal{N}_s(m) - 1}; \quad n = 1, \ldots, \mathcal{N}_s(m)$$

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$\mathcal{M} \cdot n_G \ll \mathcal{N}$ 1D RT apps. ⟶ $$\approx \sum_{n=1}^{n_G} w_{m,n} F(s_{m,n})$$

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2. Sort columns within partitions

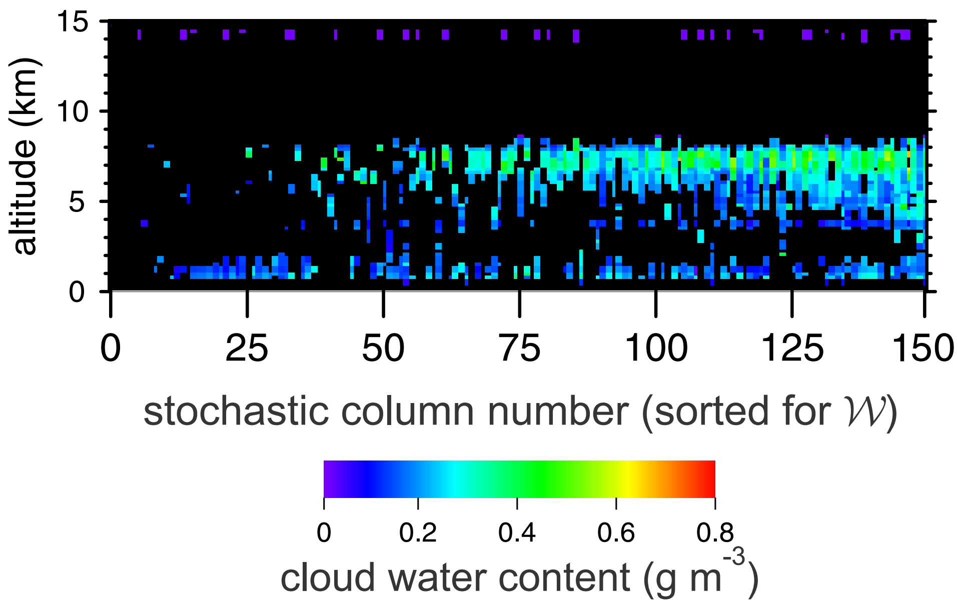
$$\frac{1}{\mathcal{N}}\sum_{n=1}^{\mathcal{N}} F_n = \frac{1}{\mathcal{N}}\sum_{n=1}^{\mathcal{N}} F_{m(n)} \equiv \int_0^1 F(s)\,ds \approx \boxed{\sum_{n=1}^{n_G} w_n F_{\widehat{s}_n}}$$

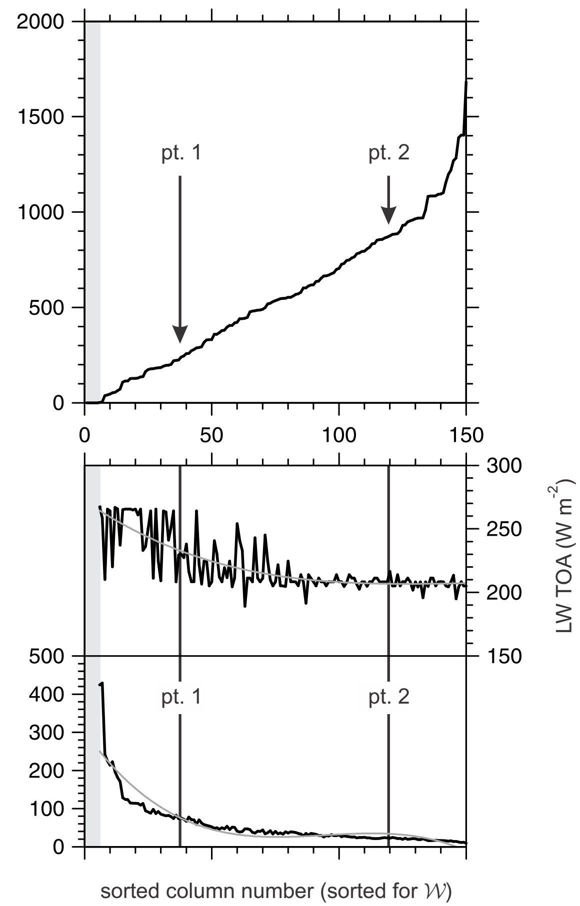
full ICA (benchmark) | sorted ICA | integral form | n_G-point GLQ McICA

$$m(1) \leftarrow \min\{W_n\} \qquad s=0 \leftarrow \min\{W_n\}$$

$$m(\mathcal{N}) \leftarrow \max\{W_n\} \qquad s=1 \leftarrow \max\{W_n\}$$

$$s_i = \frac{i-1}{\mathcal{N}-1}$$





4. Associate and distribute $F\left(s_{m,n}\right)$

Partition into sub-domains such that radiative flux profiles are "distinctive"

cf. *K*-means

** max(Q_{rad}) near cloudtops exposed-to-space

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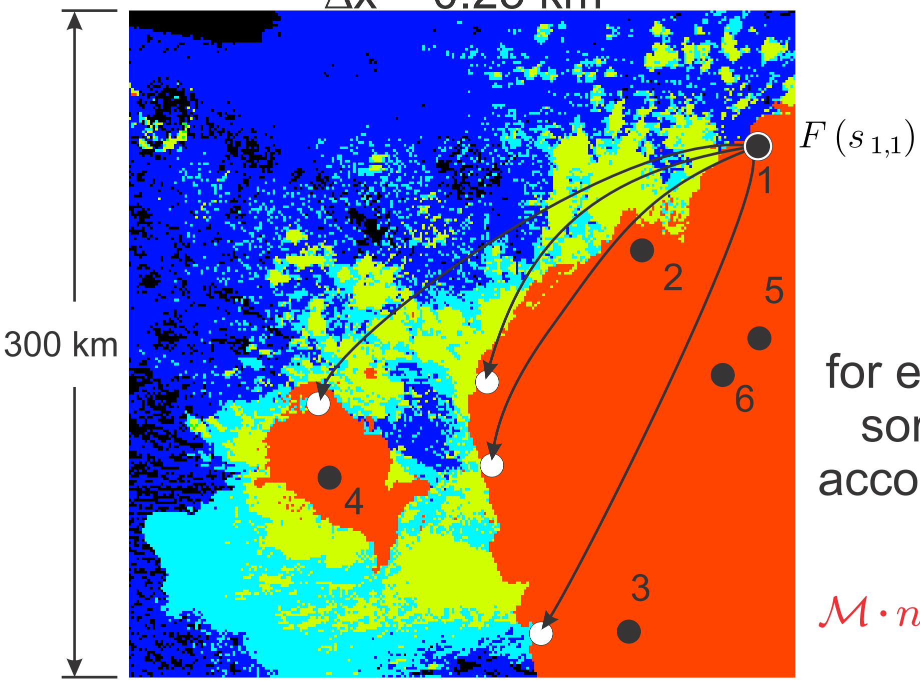

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$$= \frac{\sum_{m=1}^{\mathcal{M}} \mathcal{N}_s(m)\left[\frac{1}{\mathcal{N}_s(m)}\sum_{n=1}^{\mathcal{N}_s(m)} F_m(n)\right]}{\sum_{m=1}^{\mathcal{M}} \mathcal{N}_s(m)} = \frac{\sum_{m=1}^{\mathcal{M}} \mathcal{N}_s(m)\langle F_m\rangle}{\sum_{m=1}^{\mathcal{M}} \mathcal{N}_s(m)}$$

partitioned into $\mathcal{M}$ categories

for each partition, sort and index according to CWP

$$\begin{cases} s_{m,n} = \dfrac{n-1}{\mathcal{N}_s(m)-1}; \quad n = 1,\ldots,\mathcal{N}_s(m) \\ \langle F_m\rangle = \displaystyle\int_0^1 F(s_m)\mathrm{d}s_m \end{cases}$$

$\mathcal{M}\cdot n_G \ll \mathcal{N}$ 1D RT apps. ⟶ $\approx \sum_{n=1}^{n_G} w_{m,n} F\left(s_{m,n}\right)$

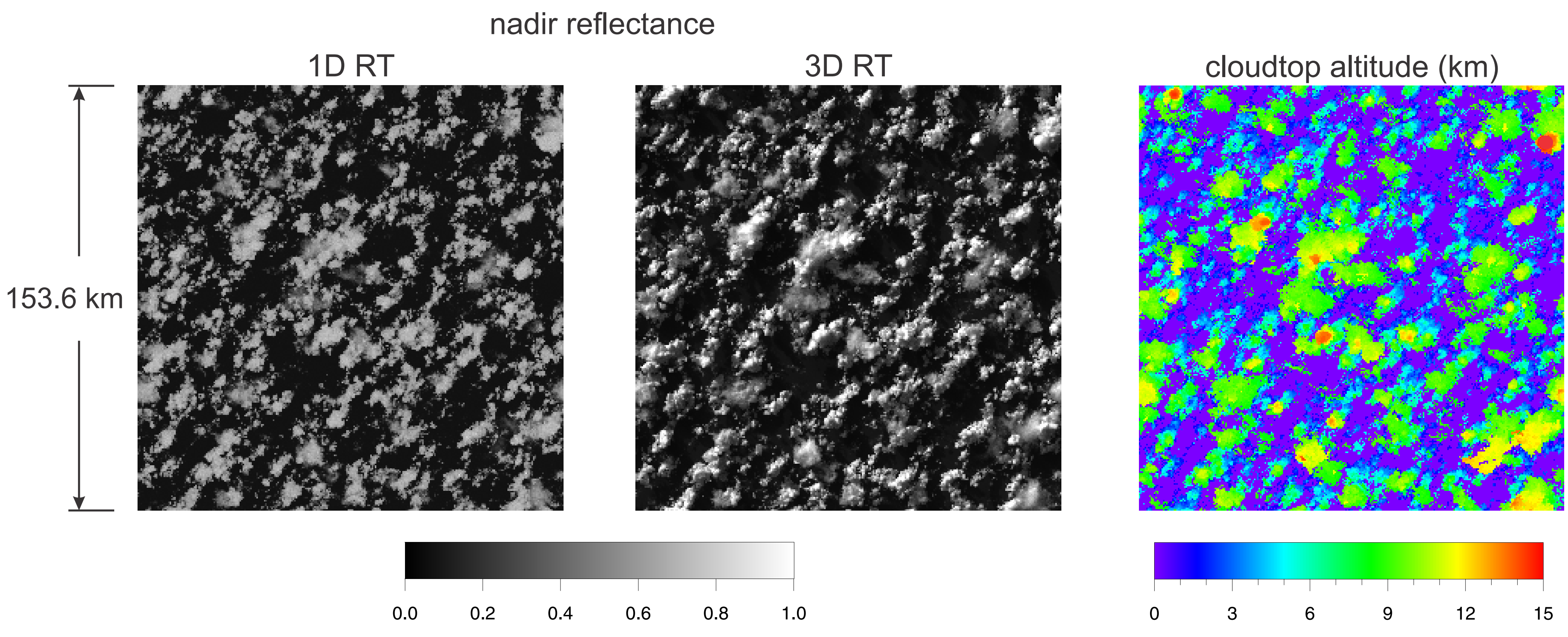
$$w_{m,n}\begin{cases} n=1 & \left[1,\ \dfrac{s_1+s_2}{2}\mathcal{N}_s(m)\right] \\ n=2 & \left[\dfrac{s_1+s_2}{2}\mathcal{N}_s(m)+1,\ \dfrac{s_2+s_3}{2}\mathcal{N}_s(m)\right] \\ \vdots & \vdots \\ n=n_G(m) & \left[\dfrac{s_{n_G(m)-1}+s_{n_G(m)}}{2}\mathcal{N}_s(m),\ \mathcal{N}_s(m)\right] \end{cases}$$

A stringent test: Deep tropical convection

M. Khairoutdinov (2005)

- 0.1 km horizontal grid-spacing
- 1536 x 1536 = 2,359,296 columns
- 76 layers from 0 to 20 km; 15 layers from 20 to 100 km
- uniform ocean surface
- $\mu_0 = 0.5$
- total cloud fraction = 0.58... 1,368,082 cloudy columns

- partition according to CTES... max(Q_{rad})

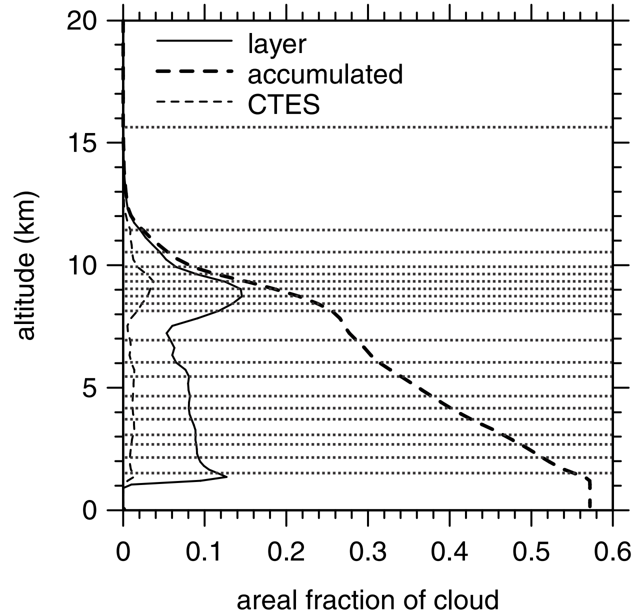


A stringent test: Deep tropical convection

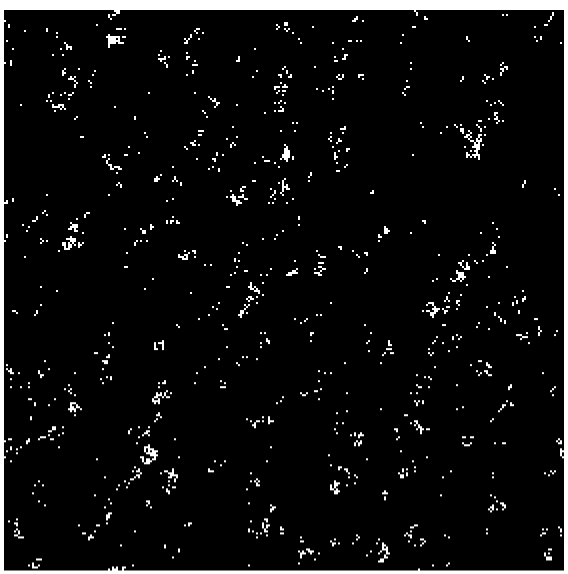
- 20 ranges of CTES altitude: (0, 1.5] km... (11.43, 15.6] km
- most cover 0.025 to 0.035 of the domain
- clear-sky is the 21st range... 990,904 columns

- range 12: cloudtops ∈ [8.127, 8.427] km
- fraction = 0.024... 55,678 columns

- sort CWP (quicksort) with (i,j) going along passively

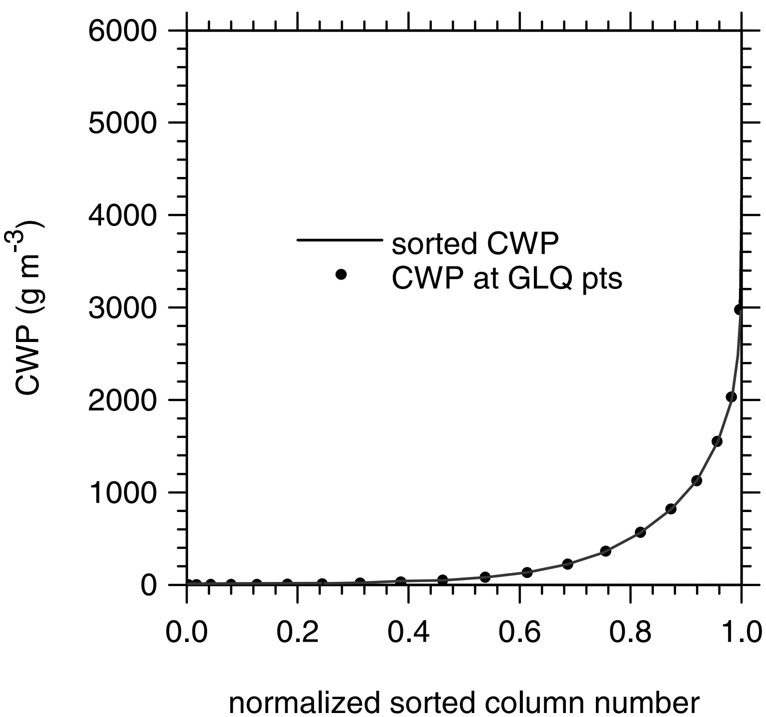


cloudtops ∈ [8.127, 8.427] km

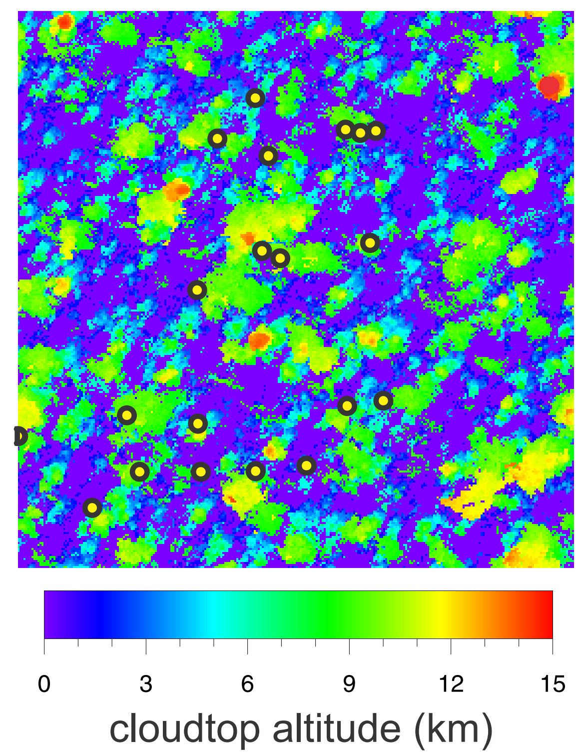


n_G = 20

cloudtops ∈ [8.127, 8.427] km



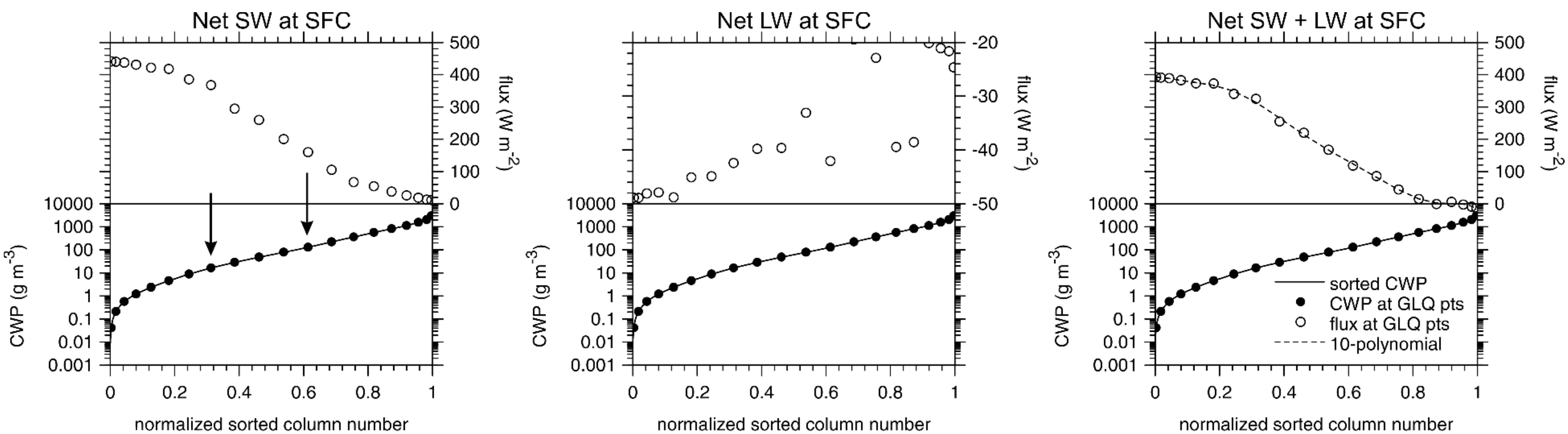
positions of 20 GLQ points



n_G = 12 of 20

cloudtops $\in$ [8.127, 8.427] km

cloudy columns in this partition = **55,678**



$$\int_0^1 f(x)\,dx \approx \sum_{n=1}^{n_G} w_n f\left(x_n\right)$$

- $f(x)$ is approximated by a poly. deg· $\leq$ ($2n_G$ - 1) on [0, 1]

- apply full BB models to each GLQ column...

n_G = 12 of 20

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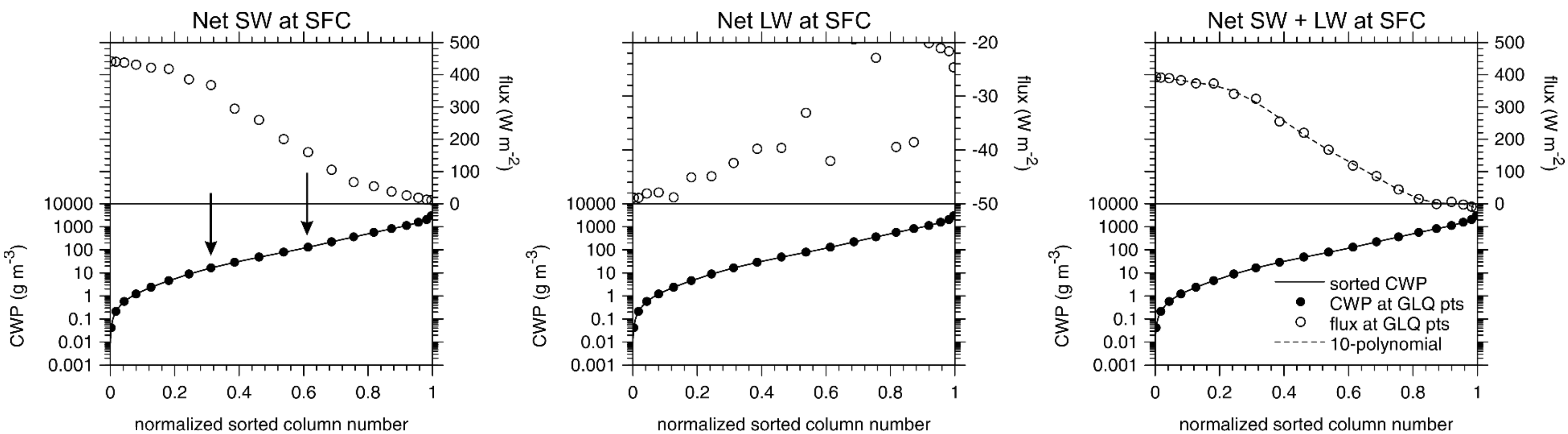

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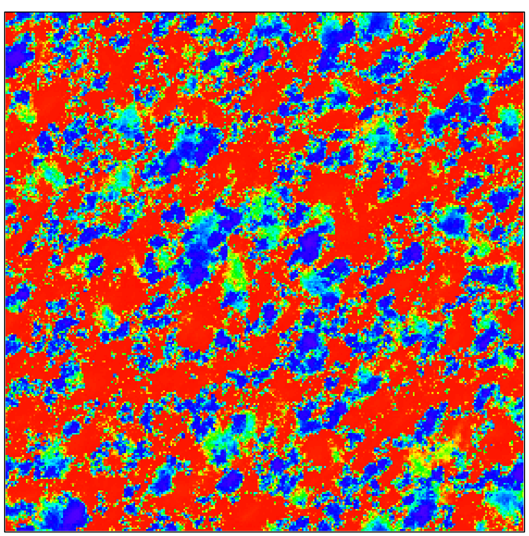
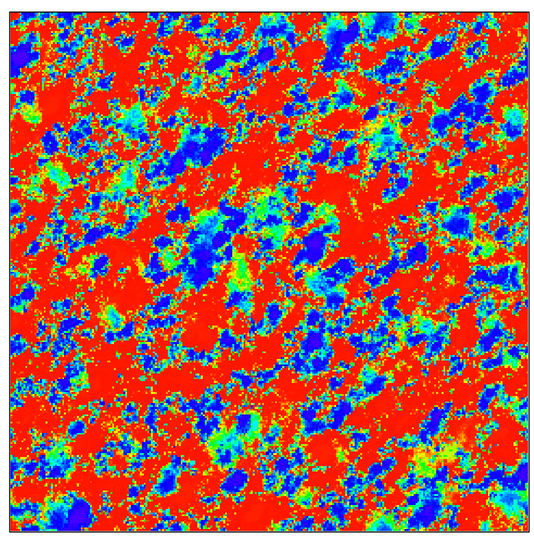
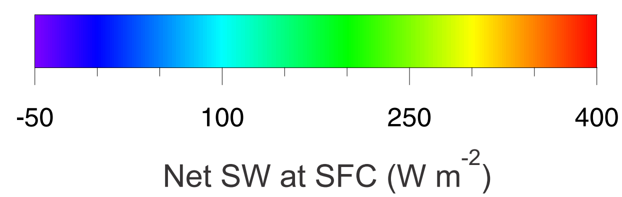
ICA



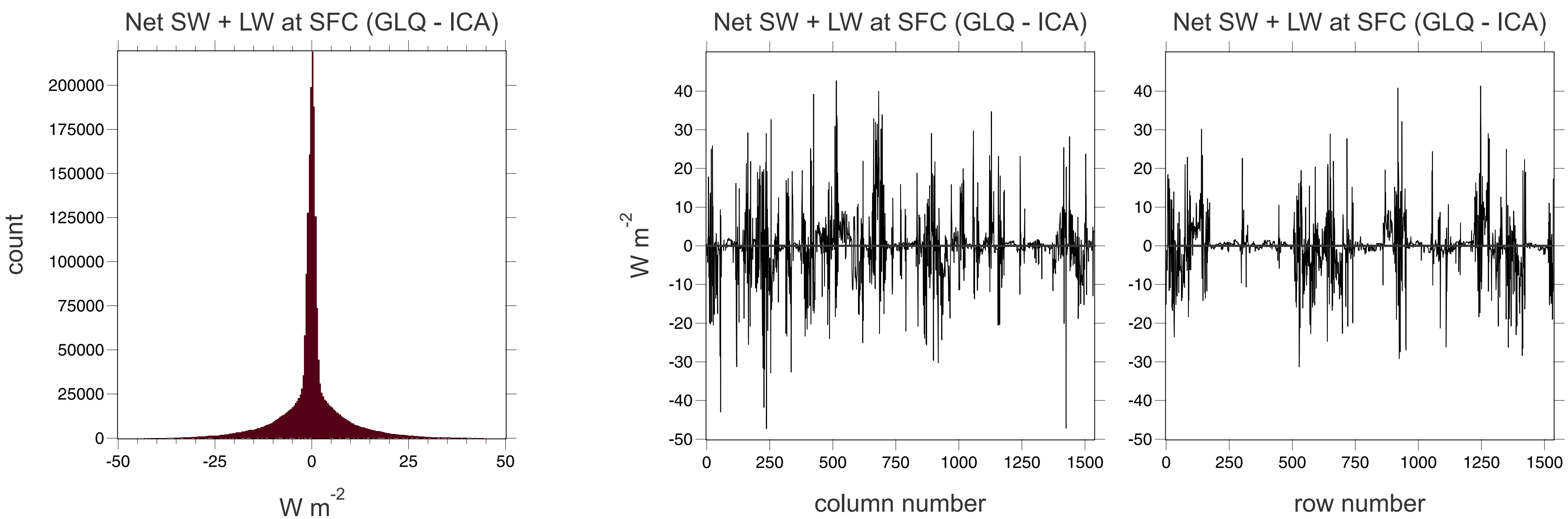
GLQ



$$\frac{1536 \times 1536}{20 \times 20\ (+\ 10)} = 5{,}754\text{x fewer}$$



A stringent test: Deep tropical convection



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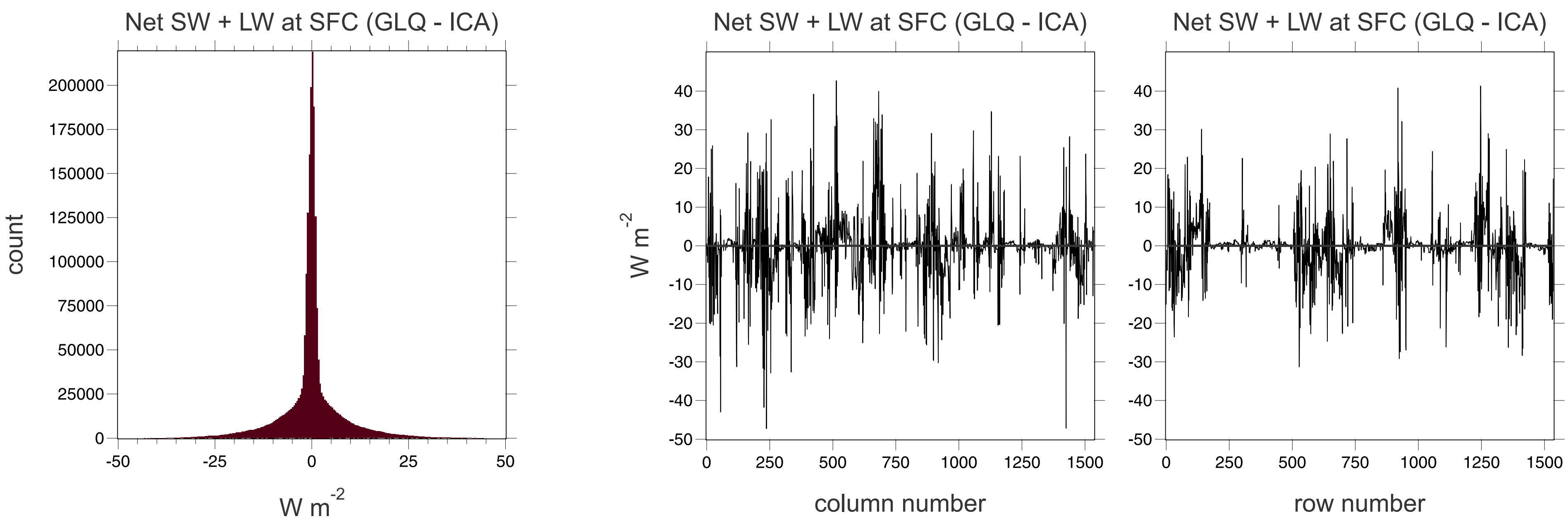


- domain avg fluxes (W m^{-2})

	cloudy-sky		all-sky	
	ICA	GLQ	ICA	GLQ
↑SW TOA	316.53	316.37	212.94	212.85
↓SW SFC	190.82	190.96	296.63	296.71
↑LW TOA	229.57	229.78	249.43	249.55
↓LW SFC	-33.10	-33.29	-40.51	-40.62
↓NET TOA	133.90	133.85	**217.63**	**217.60**
↓NET SFC	157.72	157.64	**256.06**	**256.01**

W m^{-2}

$$\frac{1536 \times 1536}{20 \times 20\ (+\ 10)} = 5{,}754\text{x fewer}$$

- application of full BB models to each GLQ <u>column</u>... but only that column...

Cloudless columns

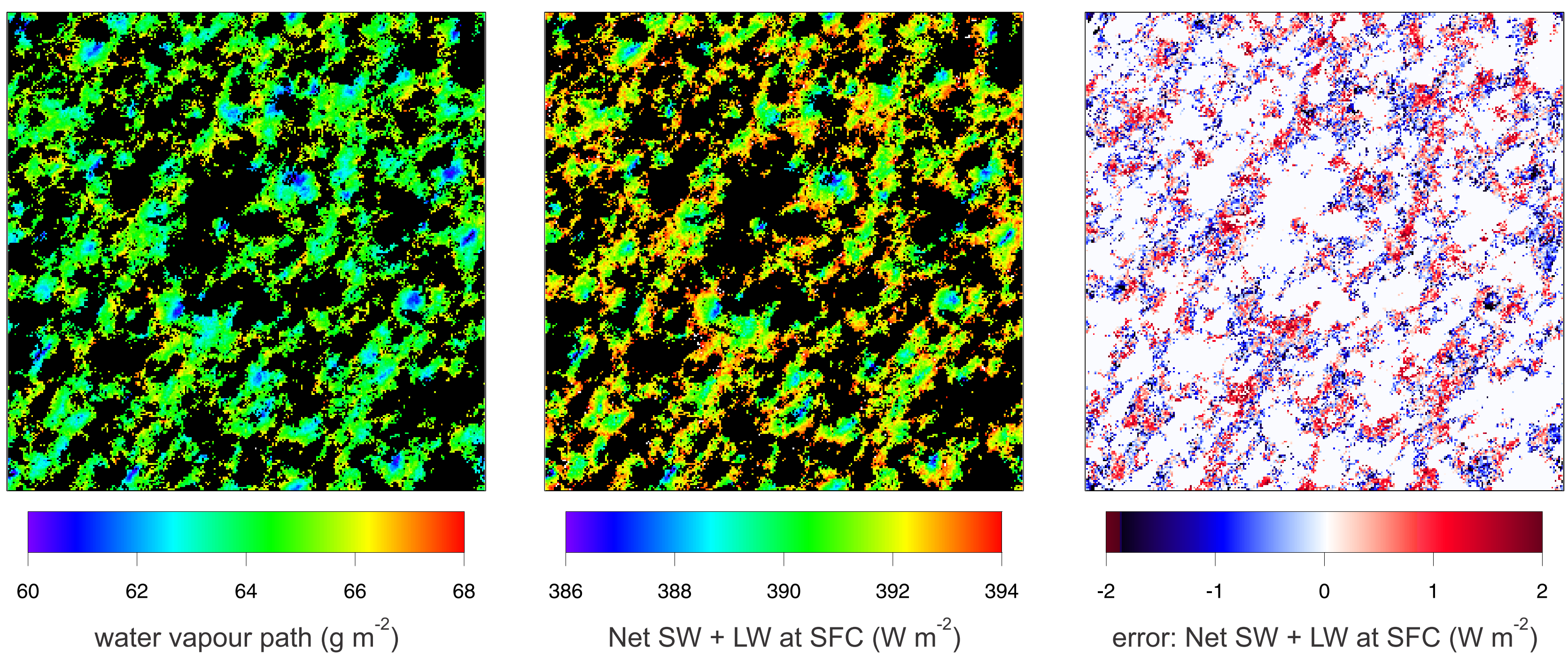
$$\frac{1}{\mathcal{N}}\sum_{n=1}^{\mathcal{N}} F_n = \frac{1}{\mathcal{N}}\sum_{n=1}^{\mathcal{N}} F_{m(n)} \equiv \int_0^1 F(s)\,ds \approx \boxed{\sum_{n=1}^{n_G} w_n F_{\widehat{s}_n}}$$

full ICA (benchmark) — sorted ICA — integral form — n_G-point GLQ McICA

$$m(1) \leftarrow \min\{W_n\} \qquad s=0 \leftarrow \min\{W_n\}$$
$$m(\mathcal{N}) \leftarrow \max\{W_n\} \qquad s=1 \leftarrow \max\{W_n\}$$
$$s_i = \frac{i-1}{\mathcal{N}-1}$$

W_n = water vapour path (g m^{-2})
n_G = 10

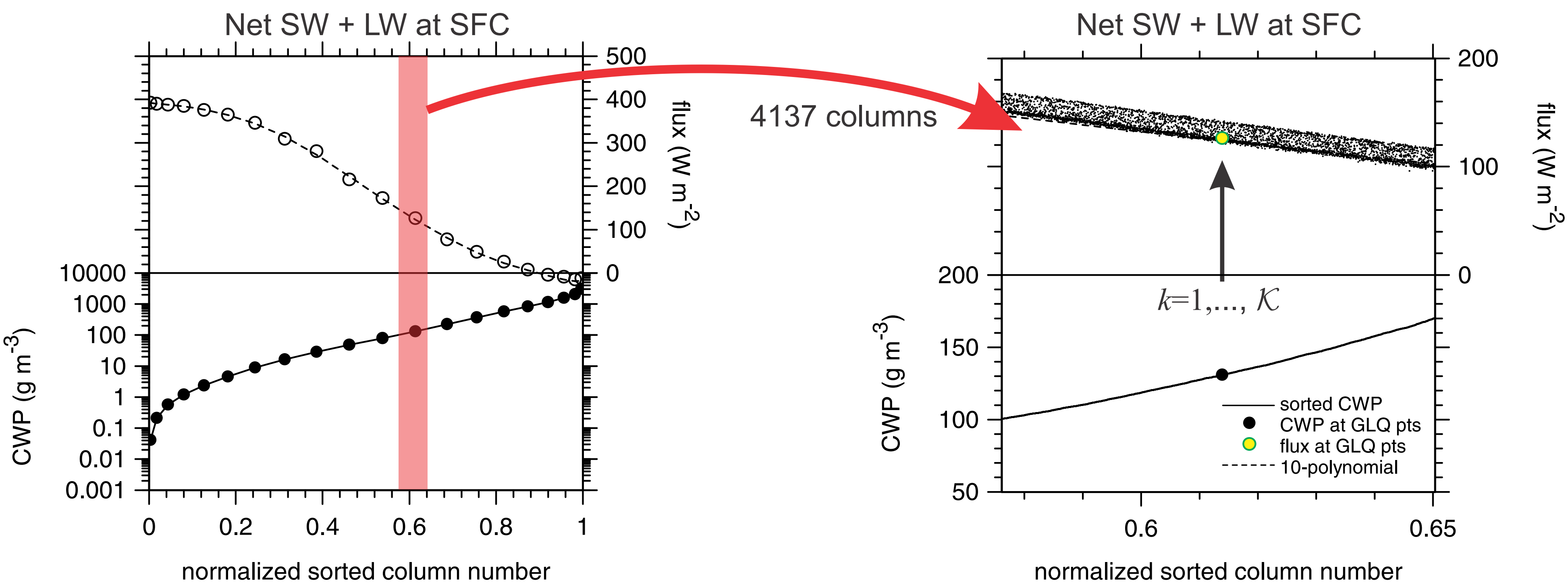
$\longrightarrow$ only 10 columns to represent 990,904!

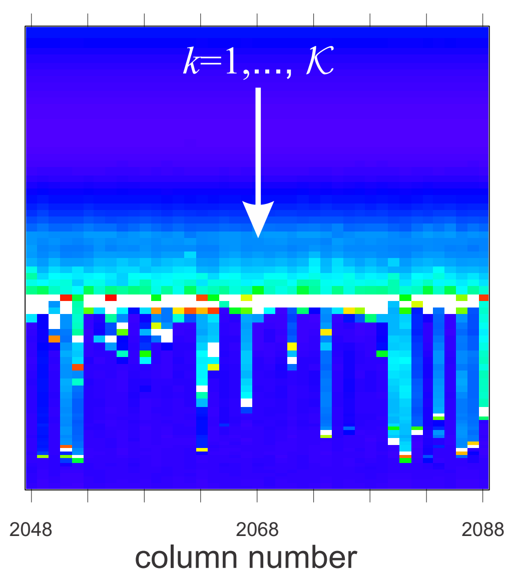


Boundary fluxes v. heating rate profiles

n_G = 12 of 20

cloudtops ∈ [8.127, 8.427] km





regular

$$\mathcal{F}_N = \sum_{k=1}^{\mathcal{K}} c_k F_{x_{GLQ(N)},k}$$

1-col McICA

Boundary fluxes v. heating rate profiles

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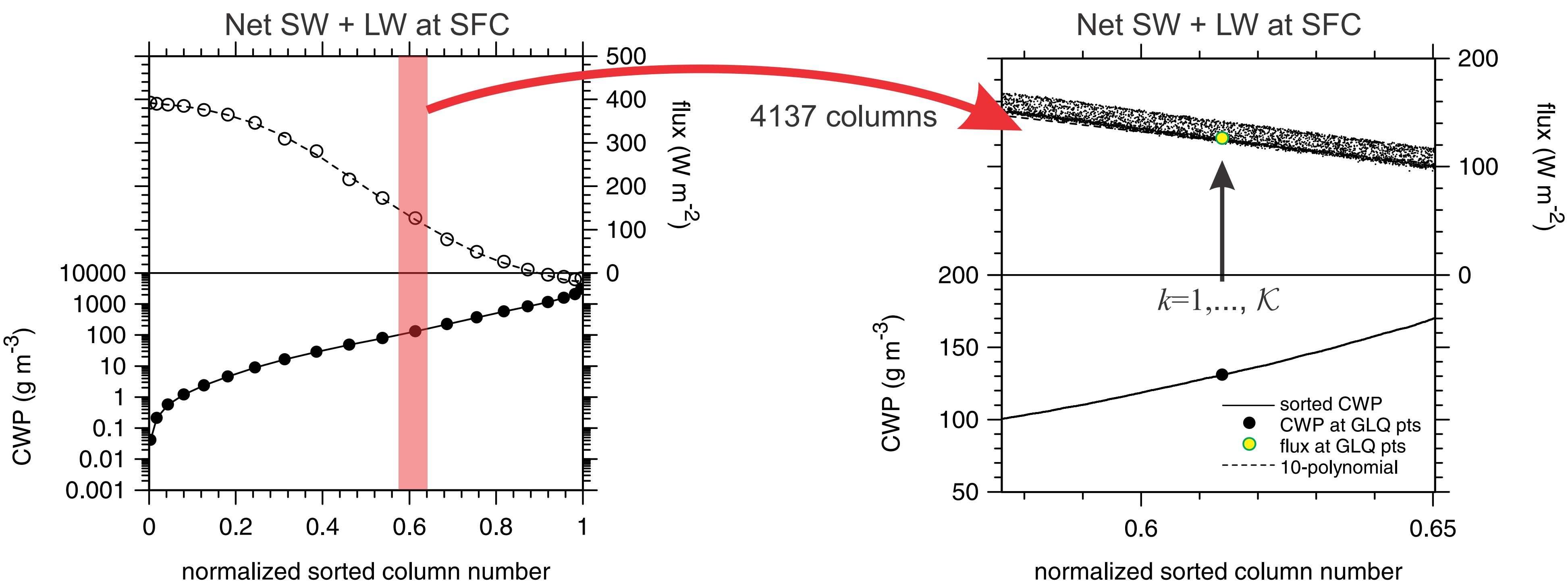
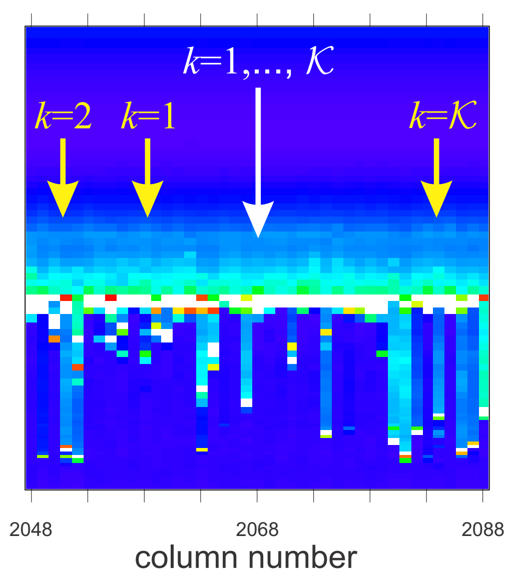




regular

$$\mathcal{F}_N = \sum_{k=1}^{\mathcal{K}} c_k F_{x_{GLQ}(N),k}$$

McICA

$$\mathcal{F}'_N = \sum_{k=1}^{\mathcal{K}} c_k F_{\widehat{x}_k(N),k}$$

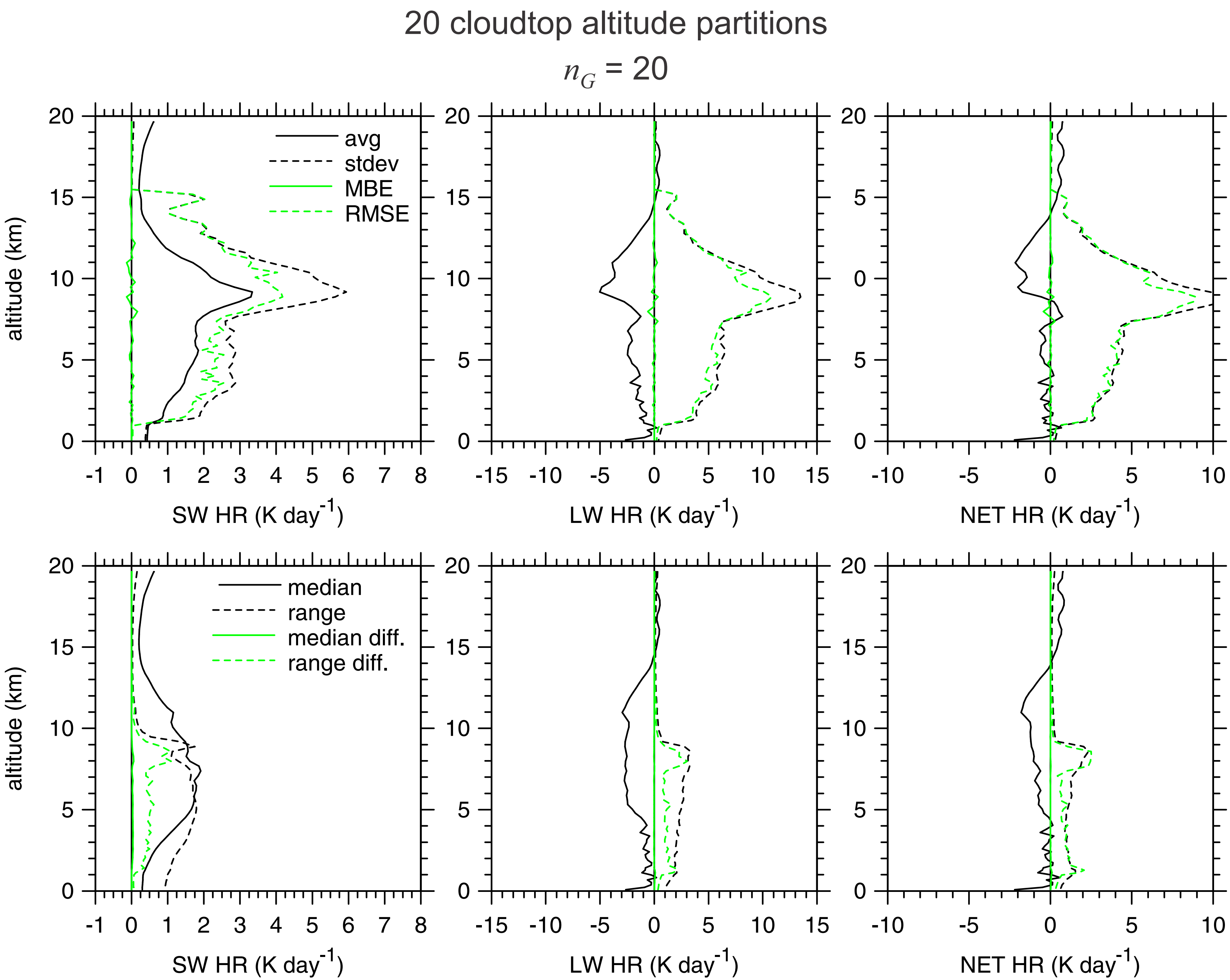
$$\widehat{x}_k(N) = x_{N-1} + (x_N - x_{N-1})\, r(k)$$

$$r(k) \in [0, 1]$$

1-col McICA ⟶ *cld* McICA

1/2 to 1/3 less noise

MBE and RMSE or quantiles?



Structure function analysis

$$S_q(L) = \langle |r\,(x) - r\,(x+L)|^q \rangle\,; \quad q \geq 0; \quad \Delta x \leq L \leq N\Delta x; \quad r_1 \leq r \leq r_2$$

$$S_2(L) \sim L^{\zeta(2)} \quad \rightarrow \quad P\,(k) \sim k^{-\beta} \quad \rightarrow \quad \zeta(2) = \beta - 1$$

- r can be either a single field or the difference between two renderings of a field

- analyses performed for surface fluxes and HRs

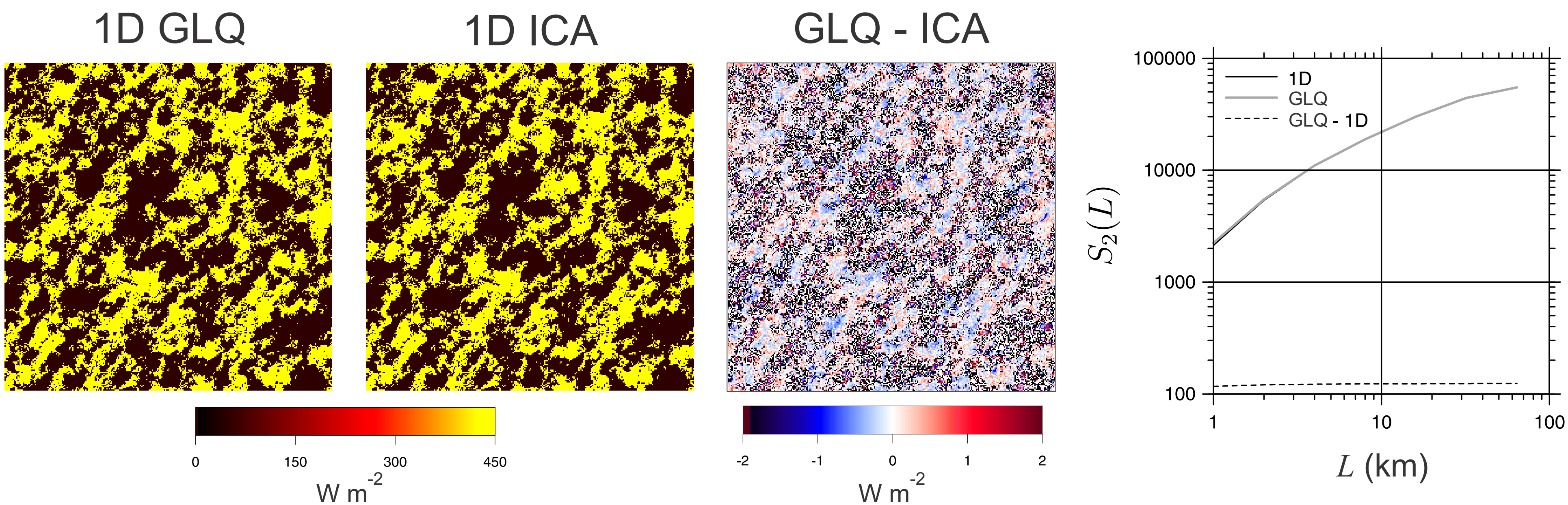
- focus on q = 2

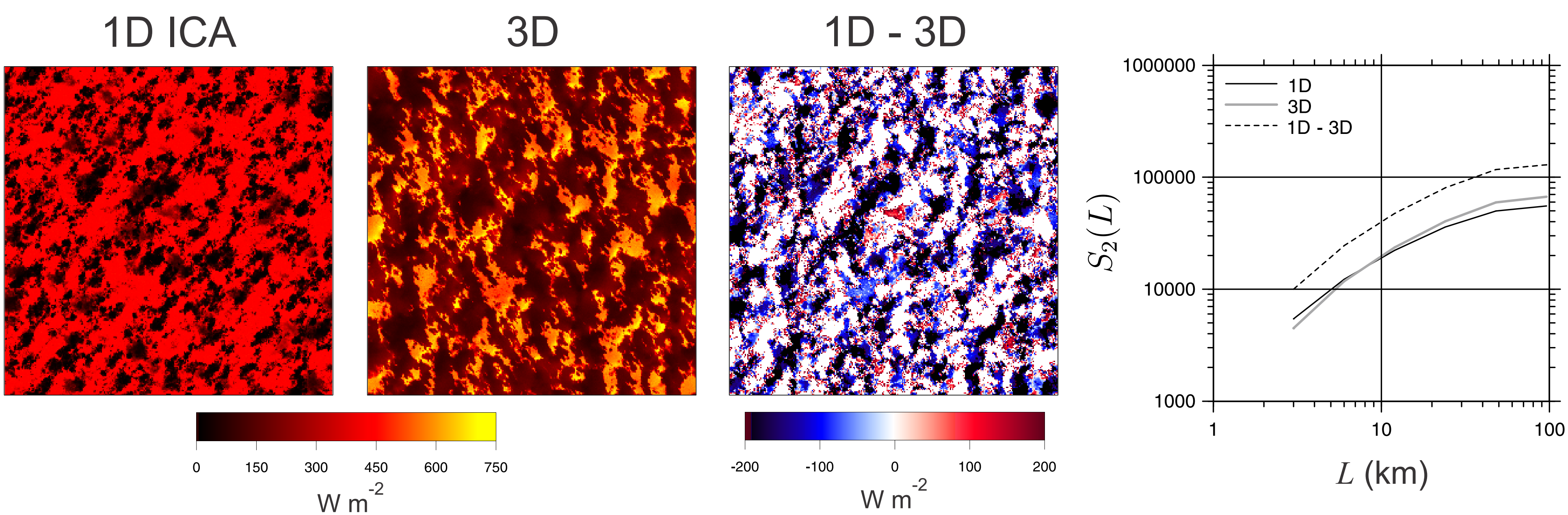
- L from 2 km to 64 km

- $\zeta(q)$ estimated by LLSR

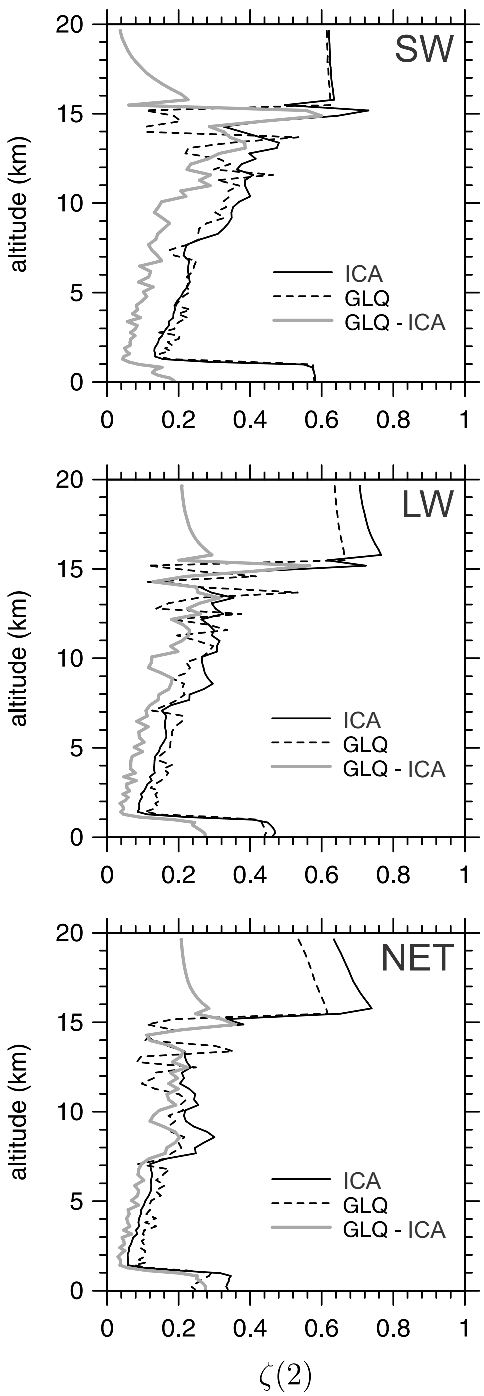
Structure function analysis

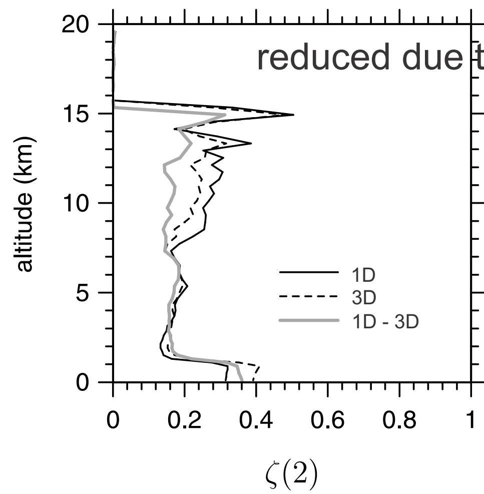
surface solar irradiance





Structure function analysis





reduced due to 1.3 x 10^9 photons

R^2 = 0.6 to 0.9

GLQ:
- ICA ≈ GLQ
- diff : closer to “noise” than fields

1D v. 3D:
- similar to 1D GLQ (degraded res. + homogenized WV)
- diff close to fields themselves

$$S_2(L) \sim L^{\zeta(2)}$$

$\zeta(2)$ at surface

	SW	LW	NET
ICA:	0.64 (0.84)	0.52 (0.84)	0.64 (0.84)
GLQ:	0.63 (0.84)	0.49 (0.84)	0.64 (0.84)
ICA - GLQ:	**0.01** (0.76)	**0.31** (0.84)	**0.04** (0.85)

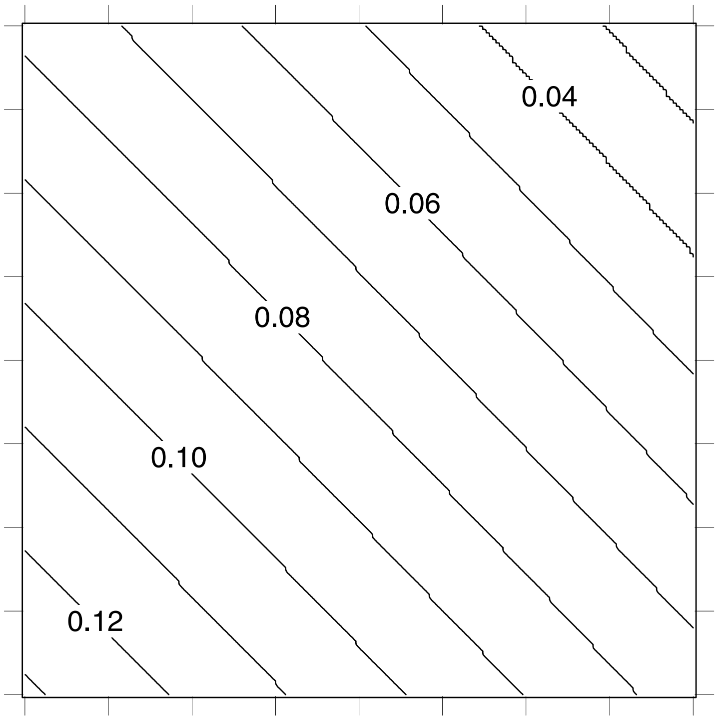
	SW
1D:	0.47 (0.60)
3D:	0.53 (0.60)
1D - 3D:	**0.51** (0.59)

Variable SW irradiance across domains

500 x 500 km @ latitude = 48°N

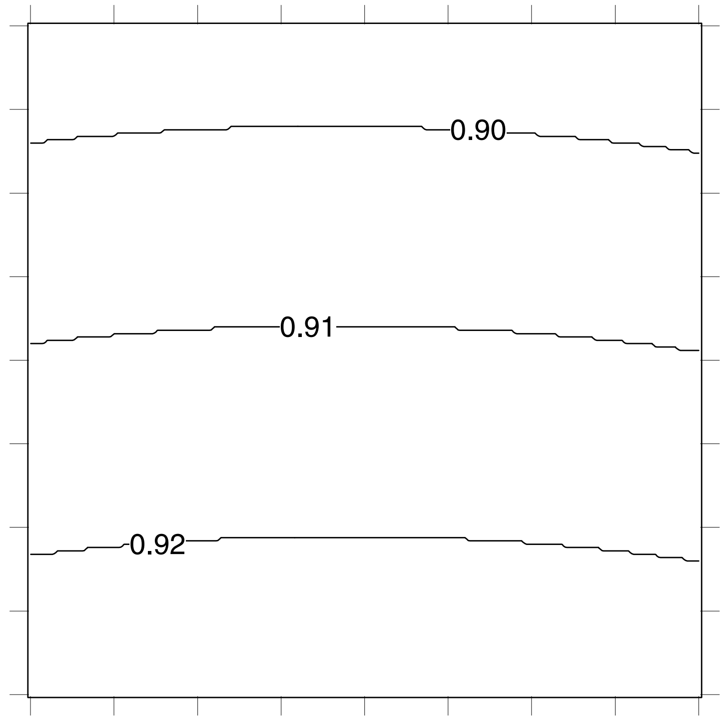
near Sunset (22-Dec)

$\langle\mu_0\rangle = 0.077$



near noon (22-Jun)

$\langle\mu_0\rangle = 0.911$

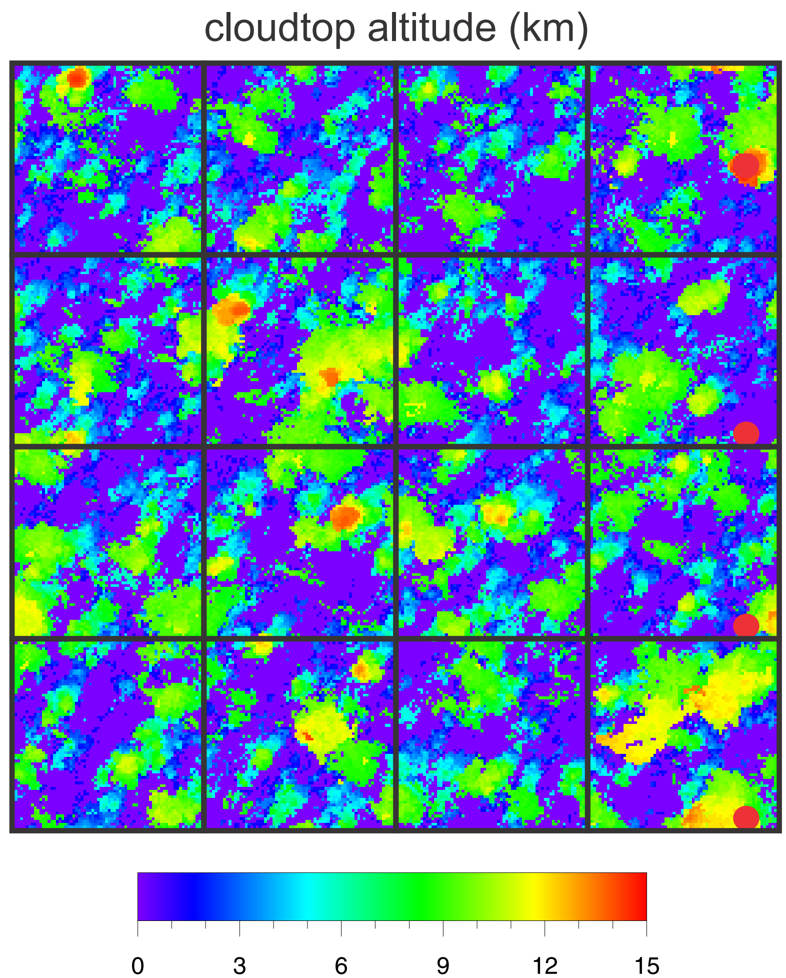


apply RT using domain-average cos(SZA) and scale when re-positioning

$$Q_{rad}(i,j) = \left[\frac{\mu_0(i,j)}{\langle\mu_0\rangle}\right] Q_{rad}(n\_GLQ)$$

Information overload

If the entire domain cannot be handled at once, sub-domains can be defined and the process applied in parallel.



- 2,359,296 columns
- reduce RT by 2,000x
- ~1,180 RT executions

1. entire domain, 20 partitions @ n_G = 59 (59 partitions @ n_G = 20)

2. 25 sub-domains, 10 partitions, n_G = 5

Variable surface conditions

- surface type: water v. land

  - use surface type as a partition and allocate n_G proportional to areas

- variable surface temperature

  - again, partition according to ranges (cf. cloudtop altitudes)

- surface elevation

  - more complicated given terrain-following vertical coordinates
  - partition according to altitude ranges followed by interpolation???

- a "partitioning algorithm"... somewhat tantamount to McICA's generator

Summary

To date

- currently RT accounts for 15% - 35% of a hi-res cloud (NWP) model's CPU time

  - RT is always 1D-ICA and usually applied with (relatively) long timesteps
  - move to 3D RT... warrant???... If not, then:

- ***proposal***: partition, sort, GLQ, and redistribute

- > 3,000x fewer calculations than full ICA... ***full-resolution*** Q_{rad} at ***every*** dynamic timestep

  - sorting and indirect accessing overheads... 1,000 - 2,000x should be possible

Ongoing activities

- *verification:* more diagnostic tests with various cloud and surface conditions

- *validation:* SAM (v6.11)... testing for a range of cloud and meteorological conditions

  - especially ones in which cloud-radiation interactions are demonstrably important

- Thank You -