Developments on unstructured Meshes

TUD

Overview of the talk

- Introduction to the Residual Distribution Framework
- Validation of RD-Schemes
- SELFE-WWMII a tightly coupled 3d-wave-current model on unstructured meshes; 1st validation runs.
- Kassandra; An operational storm surge model for the Med.
- ROMS-WWMII application in the northern Adriatic.
- Validation on unstructured WWW-III in France.
- Challenges, problems and dead ends when using unstructured meshes.
- Outlook and future tasks

Introduction to the Residual Distribution Framework (RD)

$$S_1 N_1^{n+1} = S_1 N_1^n + \Delta t \alpha_1 \Phi^T + T.F.O.C$$

$$S_2 N_2^{n+1} = S_2 N_2^n + \Delta t \alpha_2 \Phi^T + T.F.O.C$$

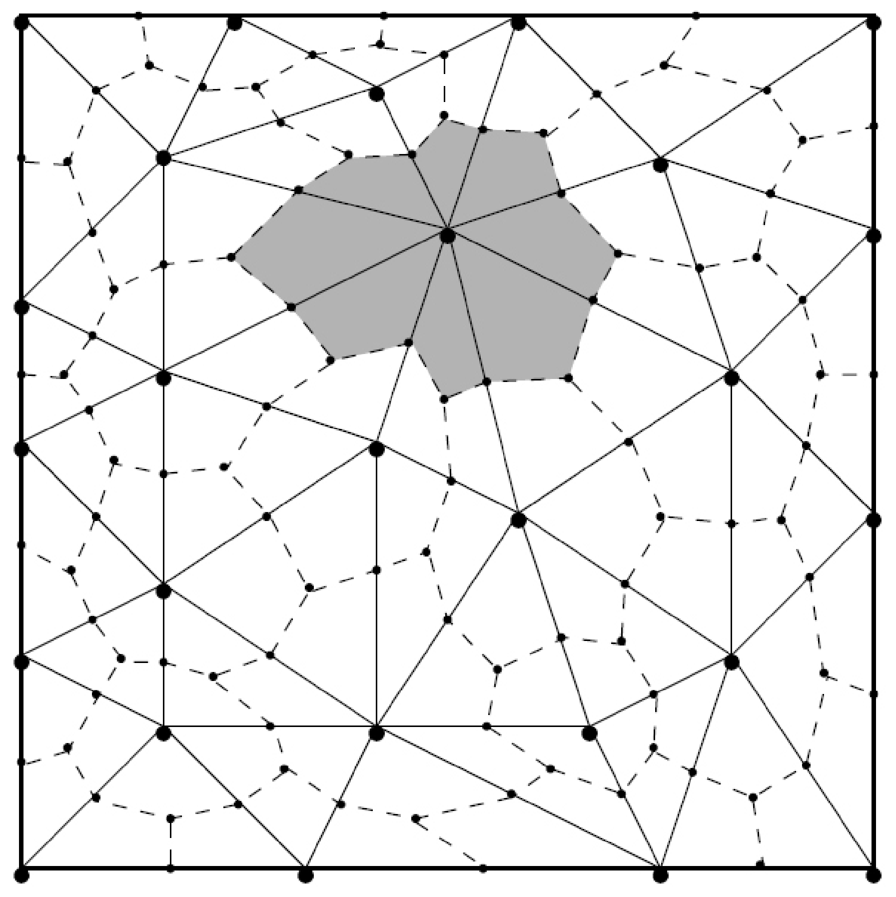
$$S_3 N_3^{n+1} = S_3 N_3^n + \Delta t \alpha_3 \Phi^T + T.F.O.C$$

$$S_i = \sum_{i=1}^{N_{con.}} \frac{1}{3} \cdot A_{N_i}$$

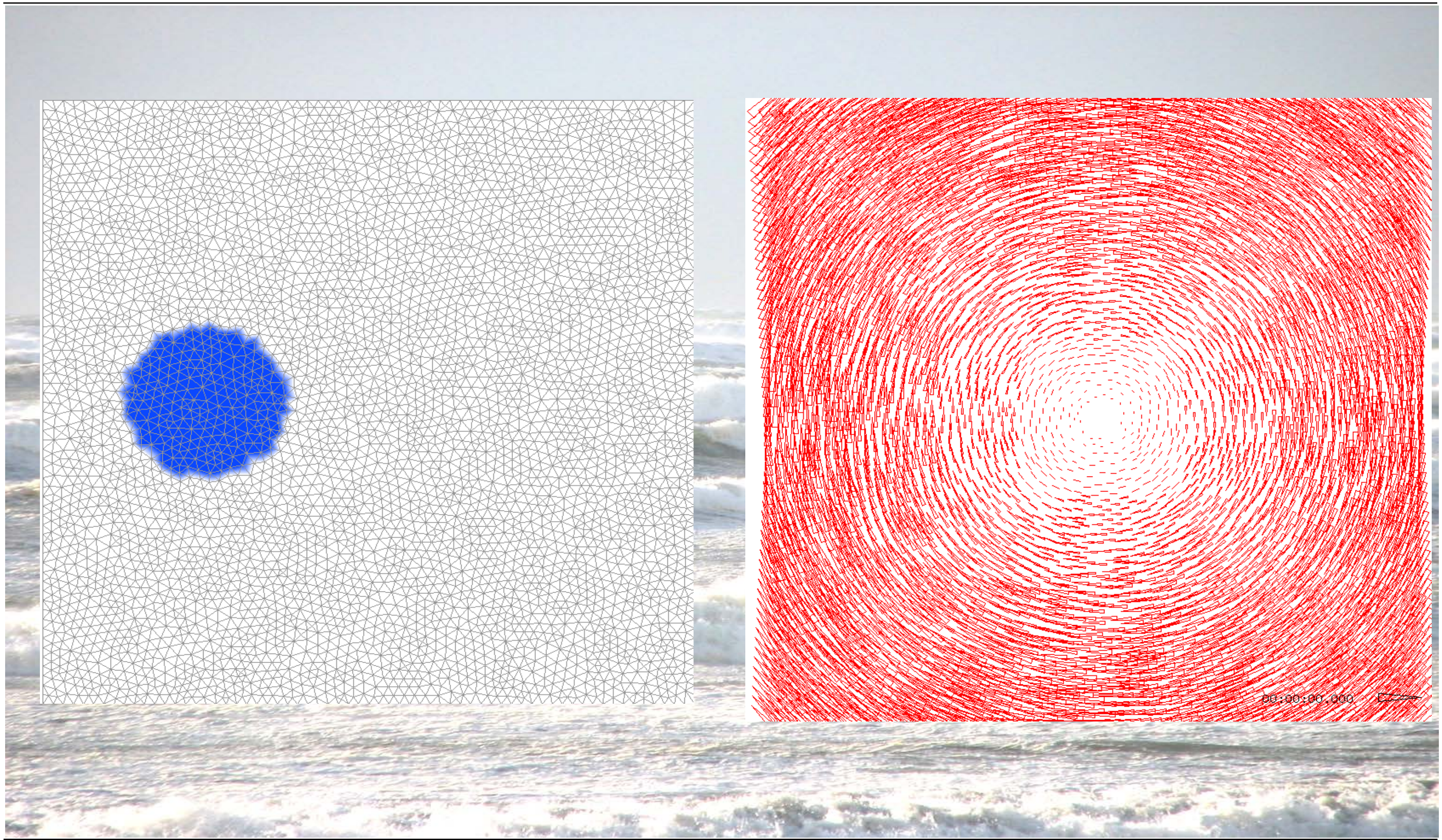
No obvious linearization of the WAE !!!

$$\Phi_T = -\oint_{\partial T} \nabla \mathbb{F} \cdot \vec{n} dS$$

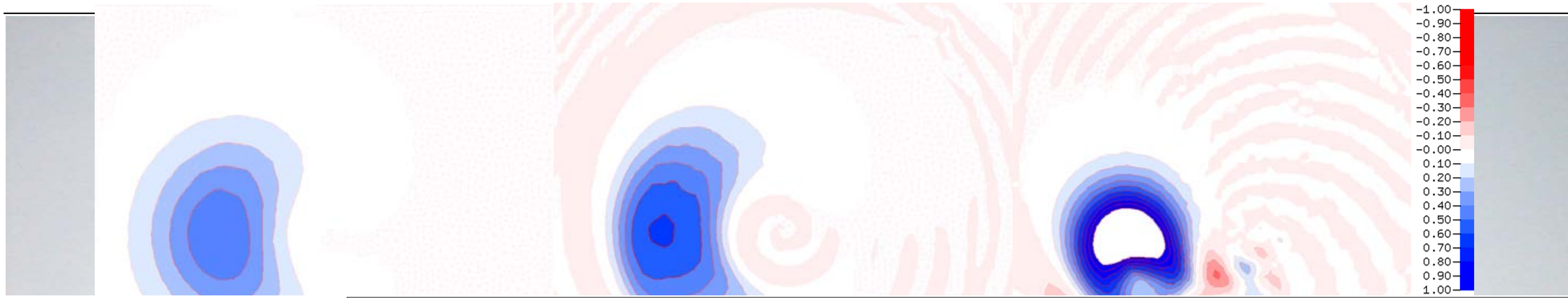
$$\mathbb{F} = \vec{c}_g \cdot N$$



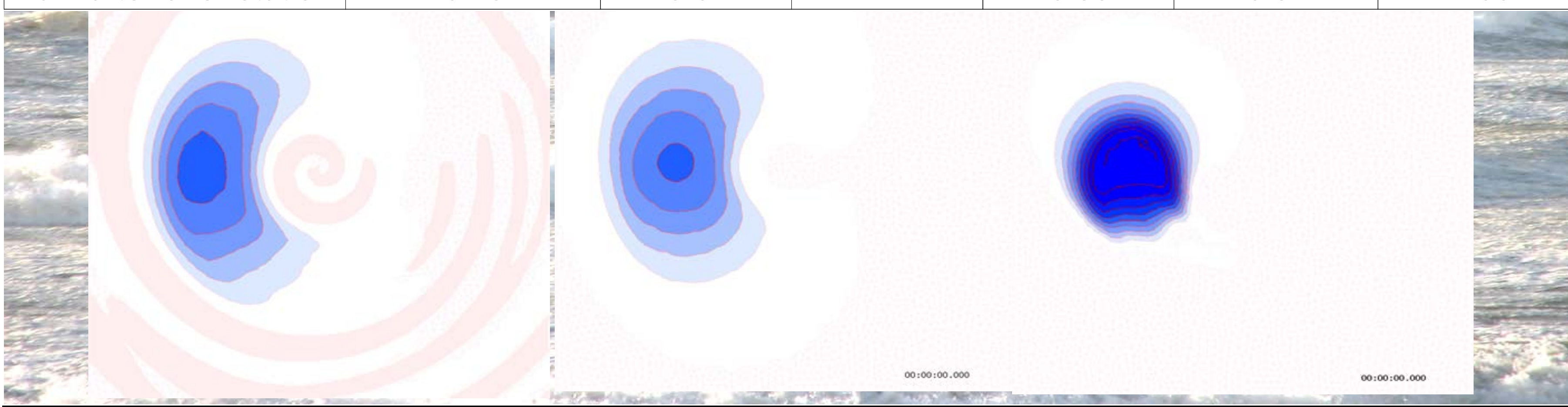
Numerical Diffusion and Dispersion Characteristics



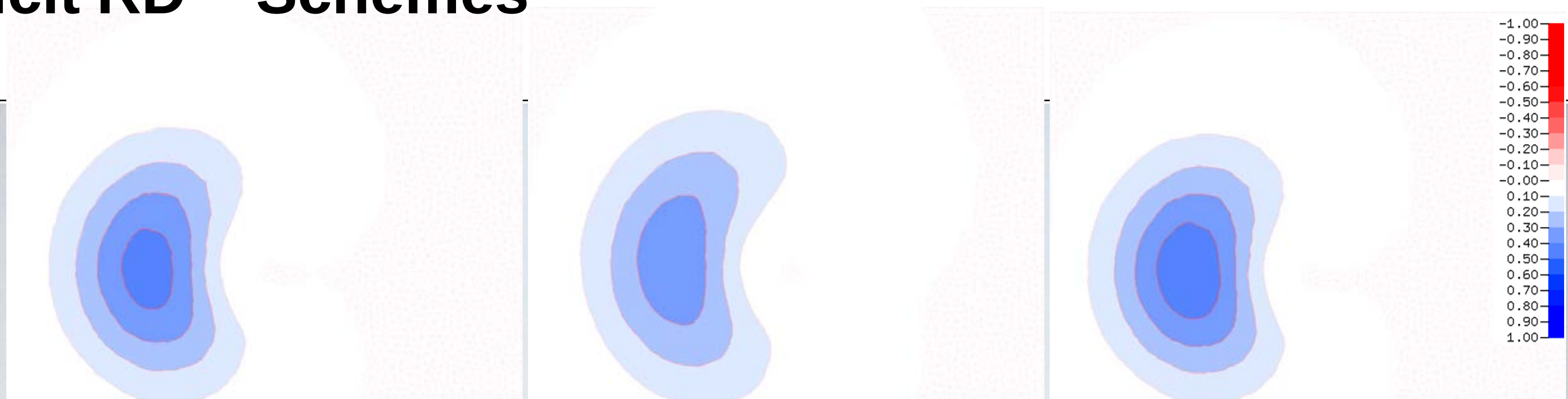
Explicit RD – Schemes



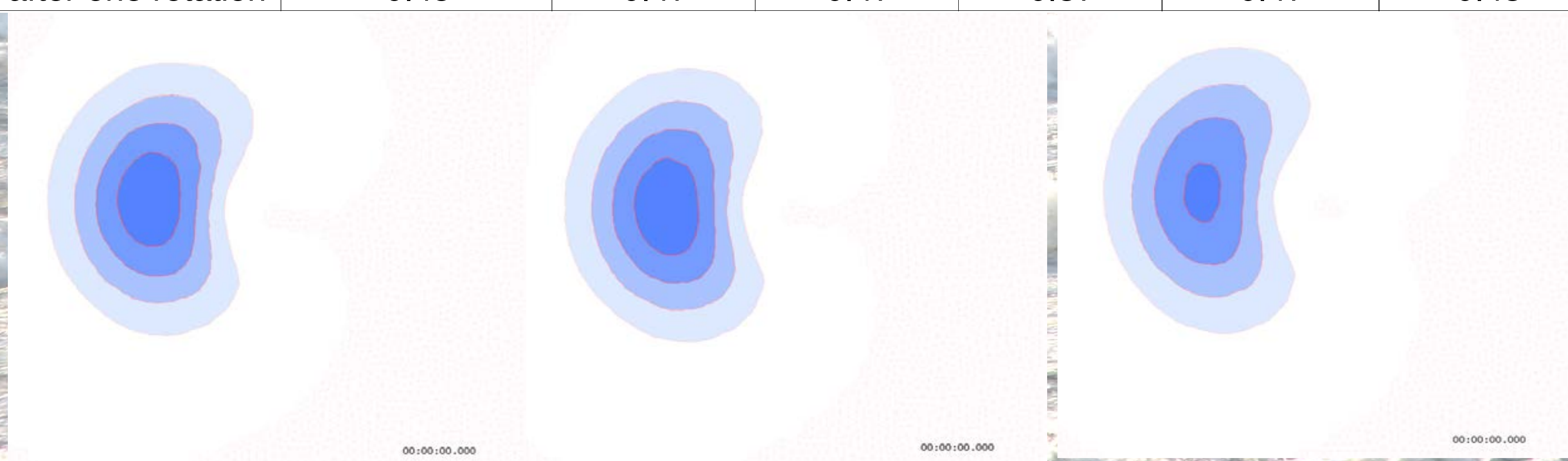
	CFL = 1.0					
Numerical Scheme	CRD-N	CRD-LDA	CRD-LAX	CRD-UCV	CRD-PSI	CRD-FCT
Min.value	0.00	-0.14	-4.00	-0.12	0.00	0.00
Max. value	1.00	1.12	1.65	1.11	1.00	1.00
Min. after one rotation	0.00	-0.06	-0.36	-0.03	0.00	0.00
Max. after one rotation	0.49	0.61	1.44	0.55	0.52	1.00



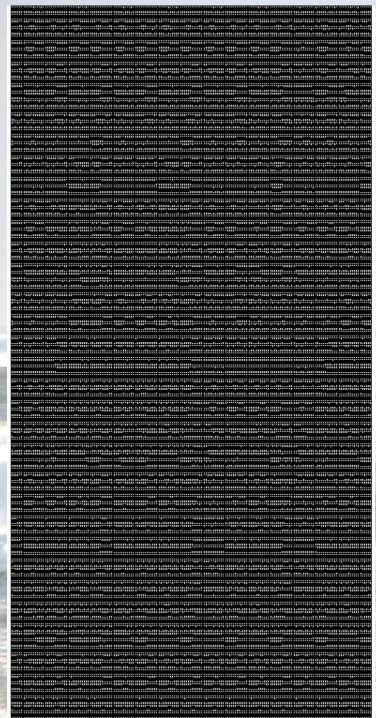
Implicit RD – Schemes

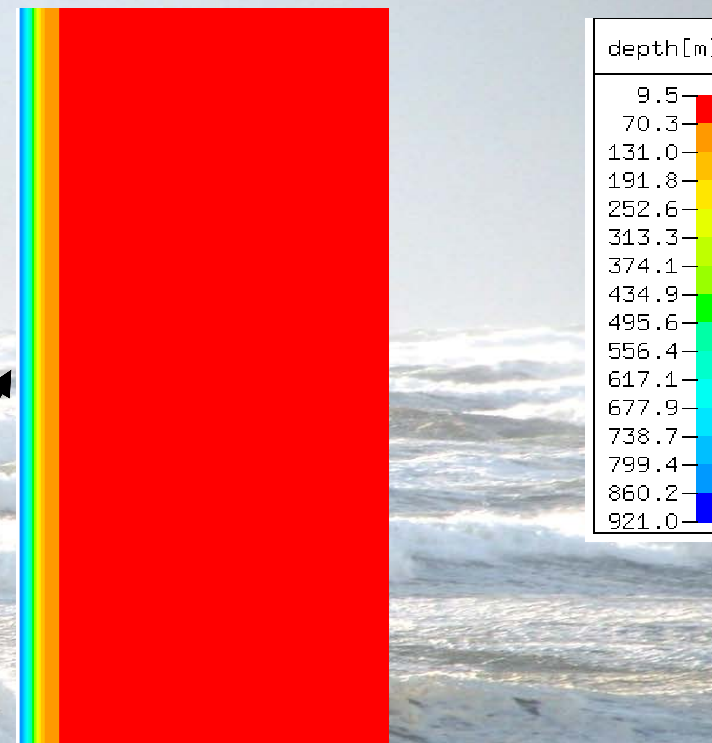


	CFL = 1.0			CFL = 5.0		
Numerical Scheme	CRD-N1	CRD-N2	CRD-N3	CRD-N1	CRD-N2	CRD-N3
Min.value	0.00	0.00	0.00	0.00	0.00	0.00
Max. value	1.00	1.00	1.00	1.00	1.00	1.00
Min. after one rotation	0.00	0.00	0.00	0.00	0.00	0.00
Max. after one rotation	0.45	0.47	0.47	0.37	0.47	0.43



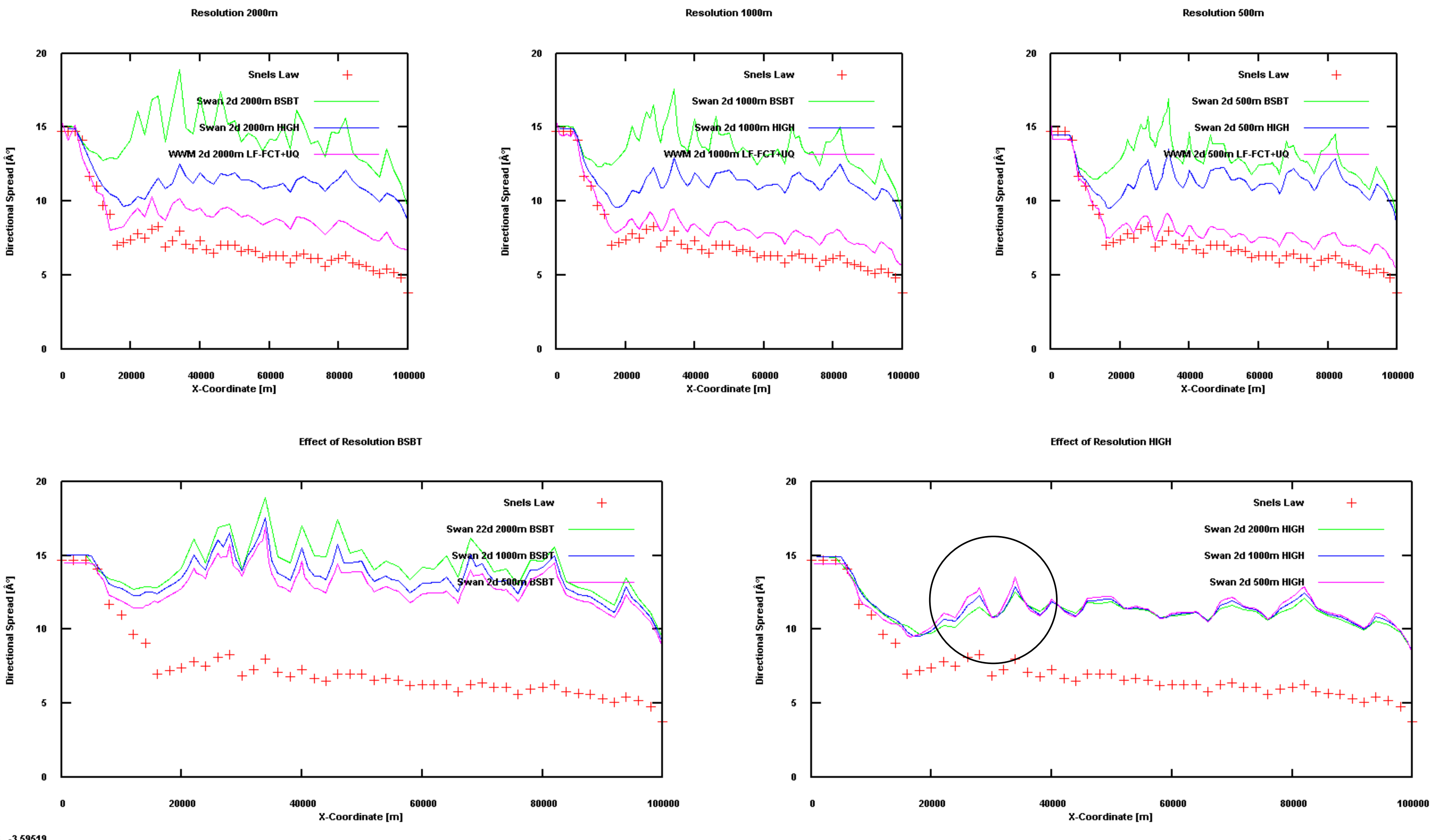
Numerical Diffusion: Continental Shelf Test Case
Ardhuin & Herbers 2005



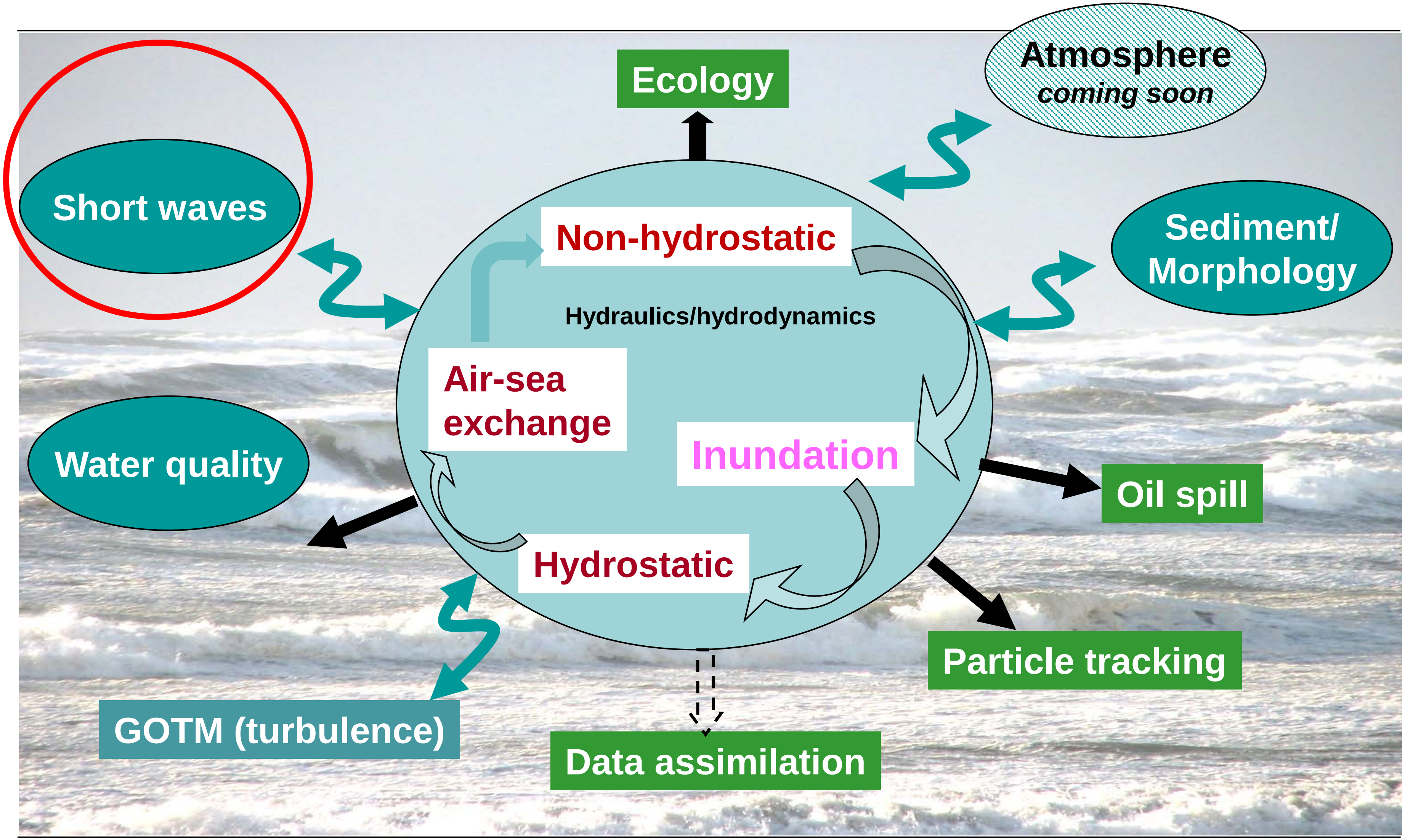


Wave spectrum at the boundary 13.88s period, 15° directional spreading, 60° incoming wave angle. Grid resolution in cross-shore direction is 2000m, 1000m and 500m for spectral space UQ-3rd order schemes have been used.

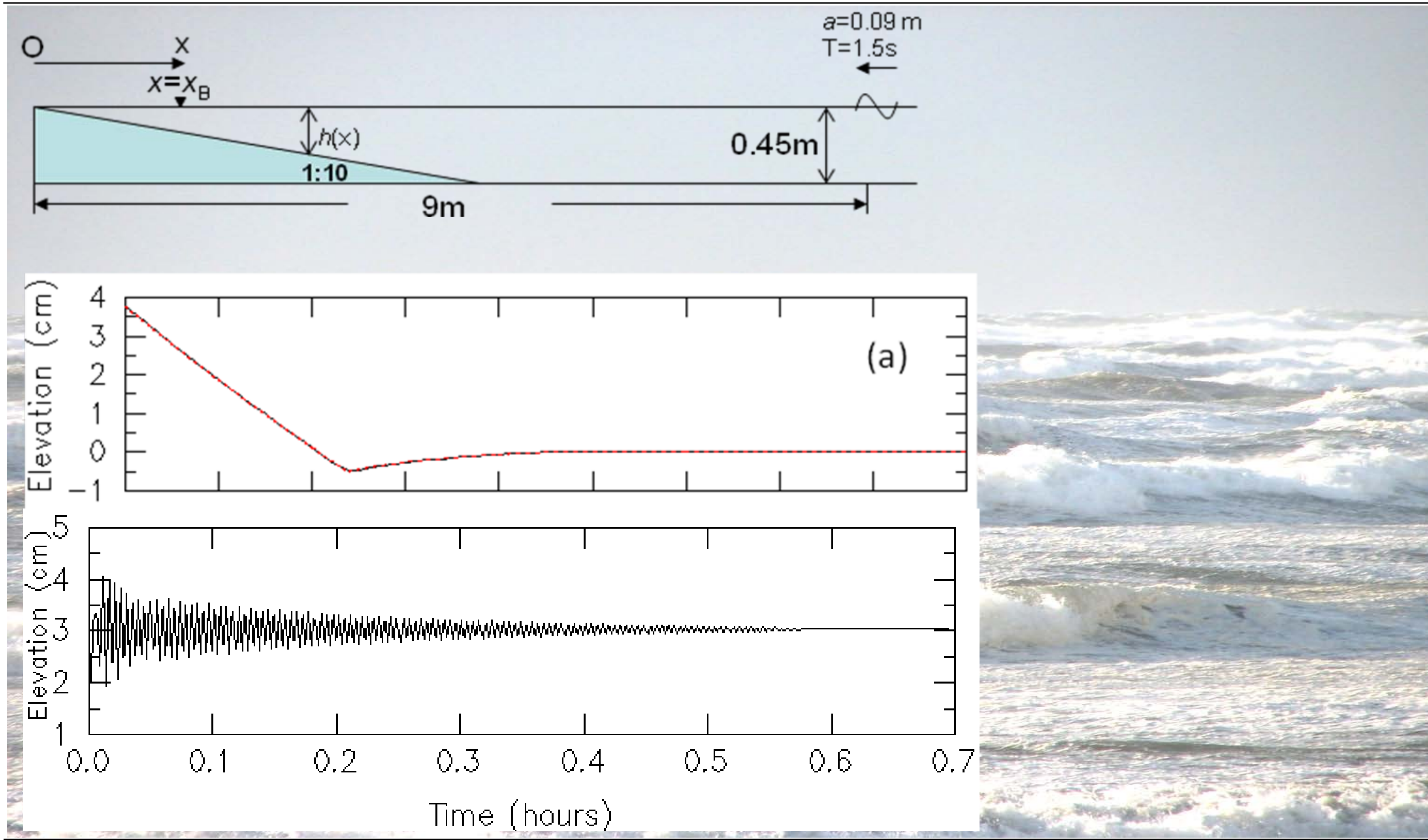
Directional resolution 5°



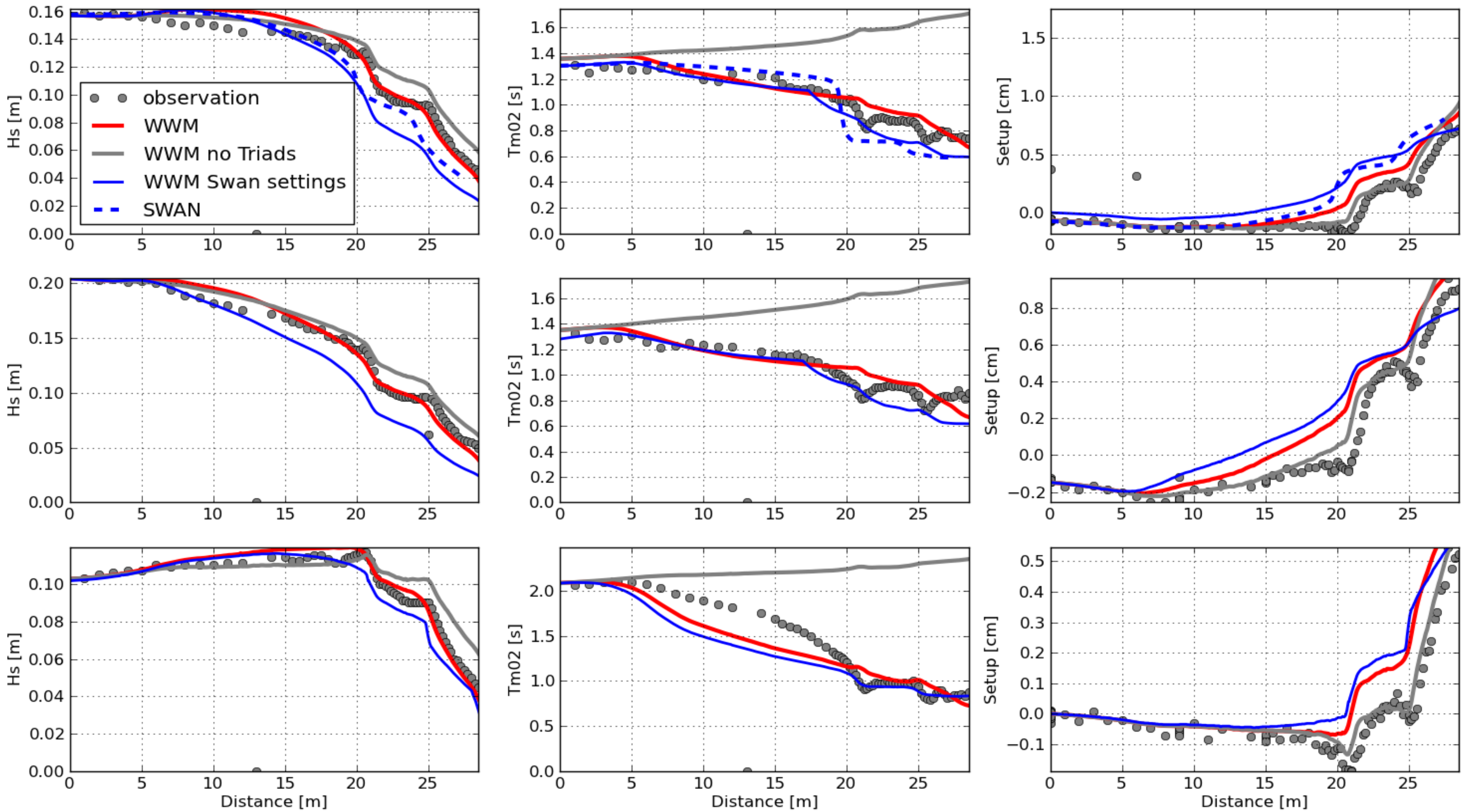
SELFE-WWMII a tightly coupled 3d-wave-current model on unstructured meshes; 1st validation runs



Validation SELFE-WWMII – Analytical Shoaling

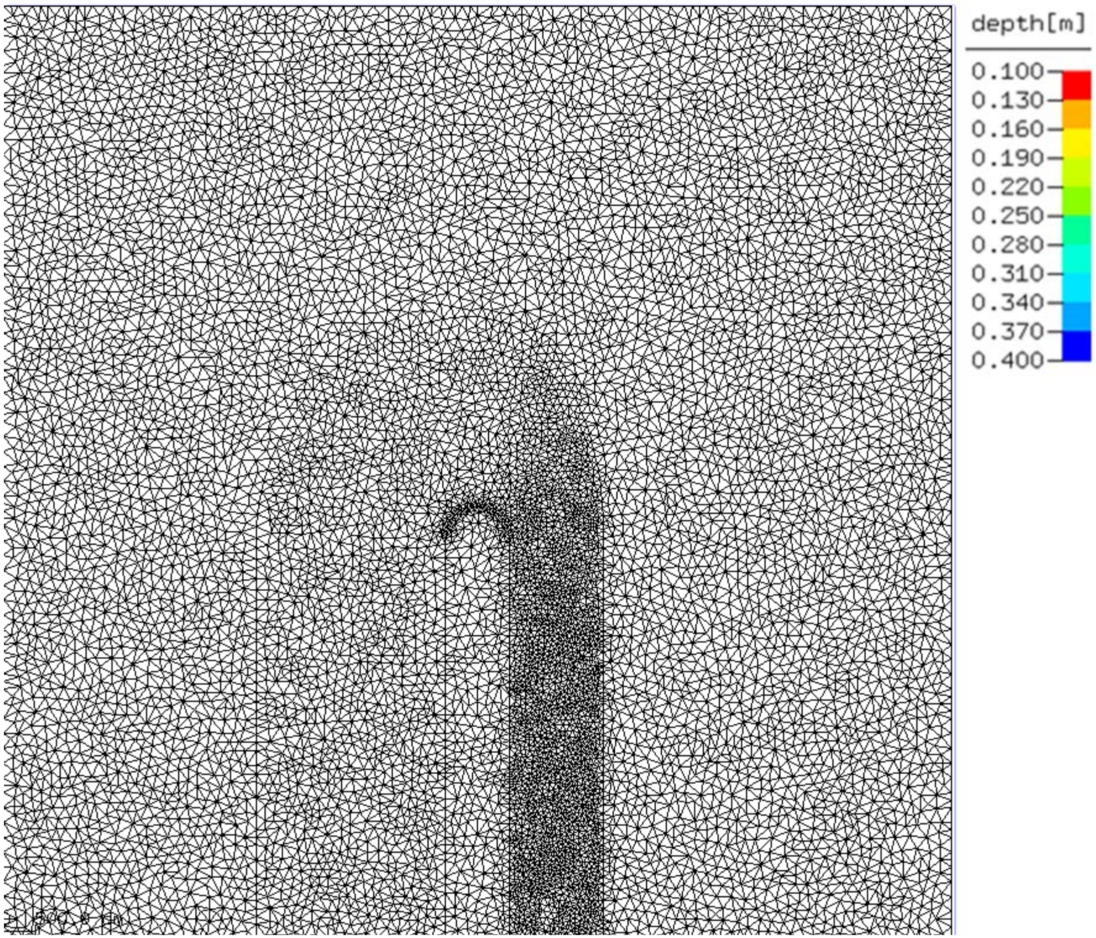


Validation SELFE-WWMII – Boers Setup Experiment



Roland, A., Zhang Y.J., Wang H.V., Meng Y., Teng Y-C, Maderich, V., Brovchenko, I., Dutour-Sikiric, M. and Zanke, U., 2012, A fully coupled 3D wave-current interaction model on unstructured grids, submitted to JGR-Oceans

Validation SELFE-WWMII – HISWA Tank



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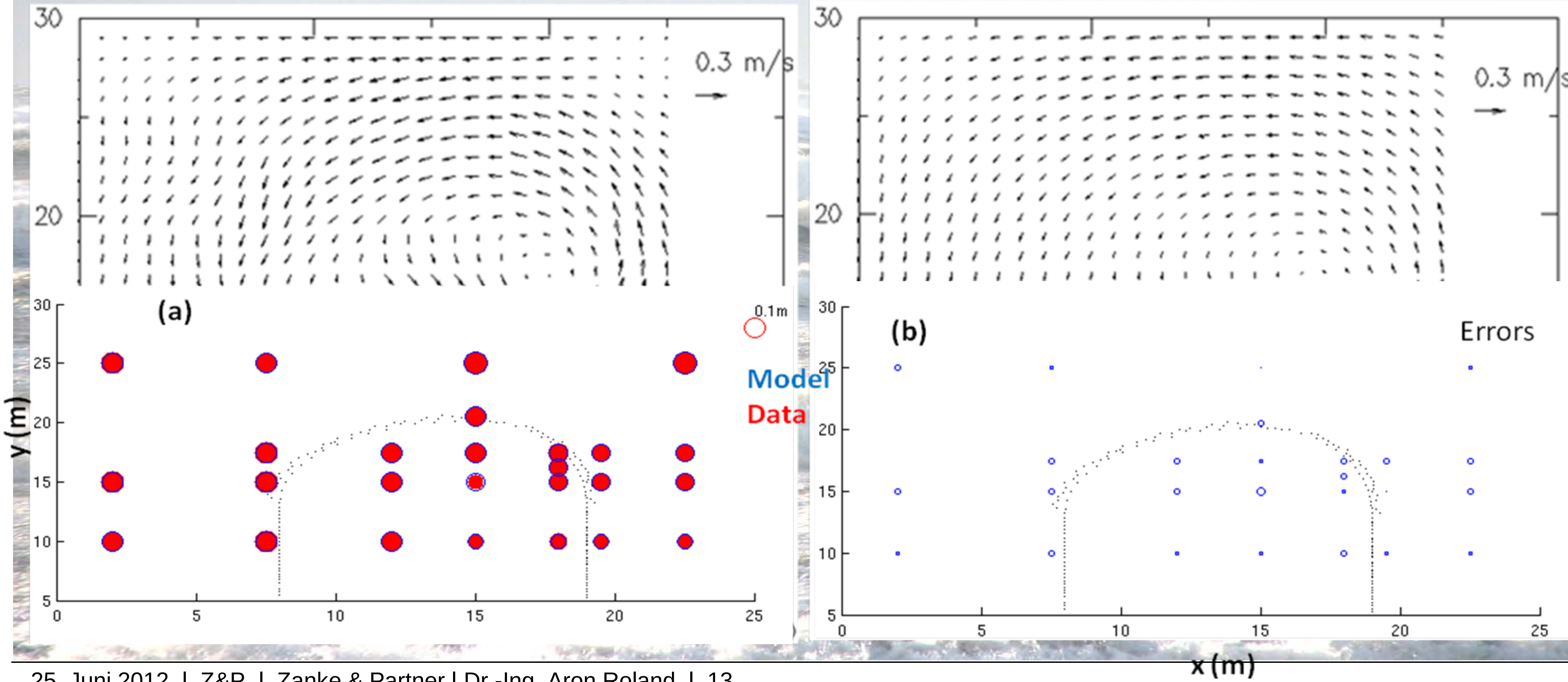
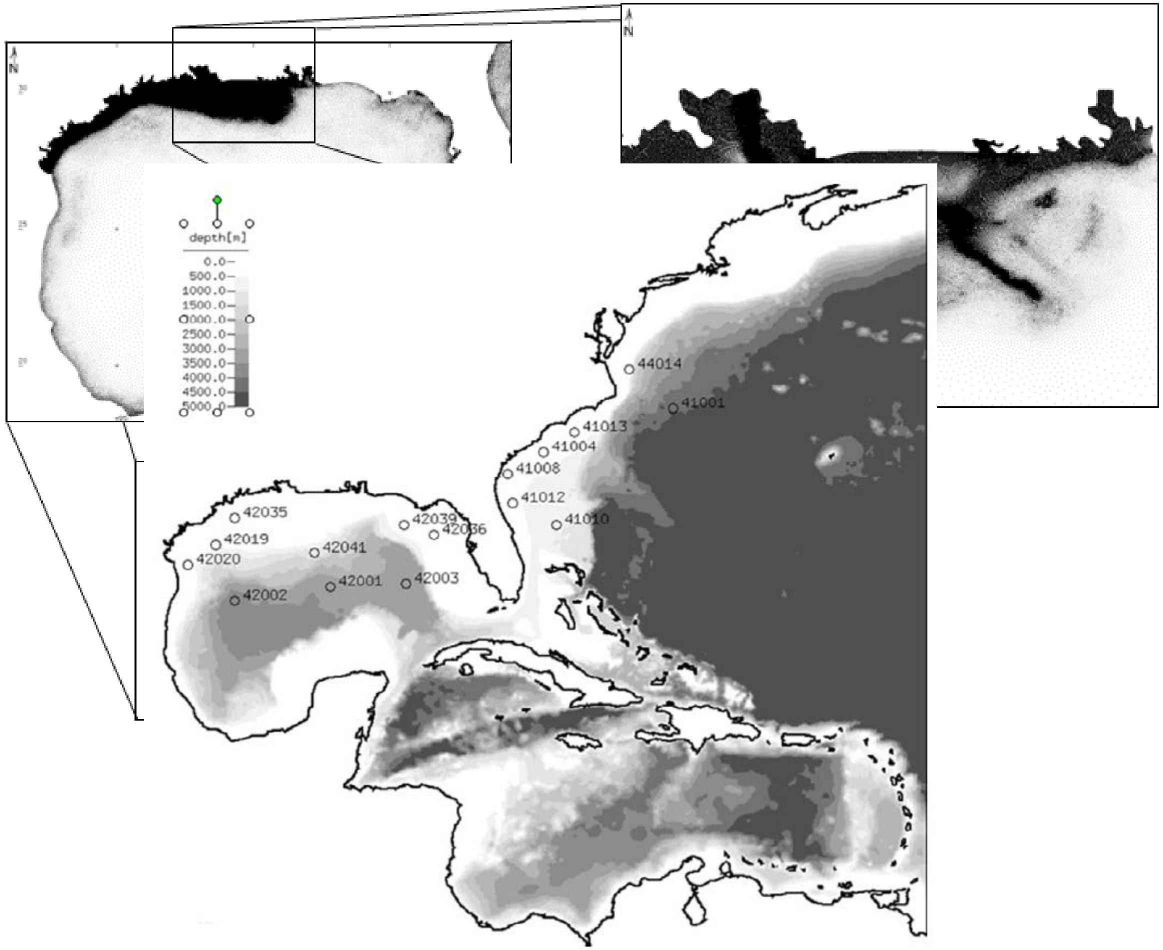
Validation SELFE-WWMII – HISWA Tank

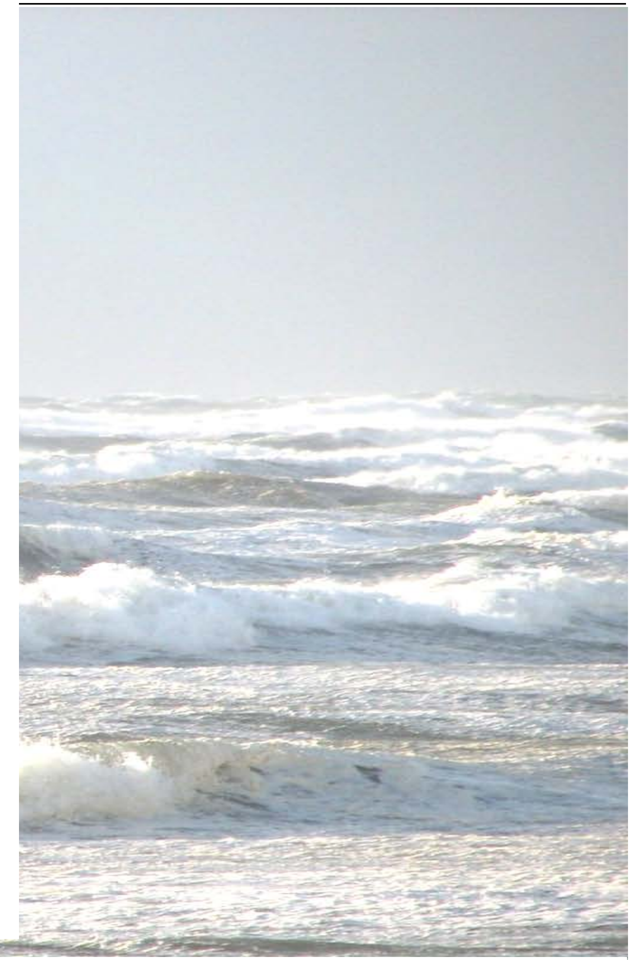
Left: Model

Right: Observation



Validation SELFE-WWMII





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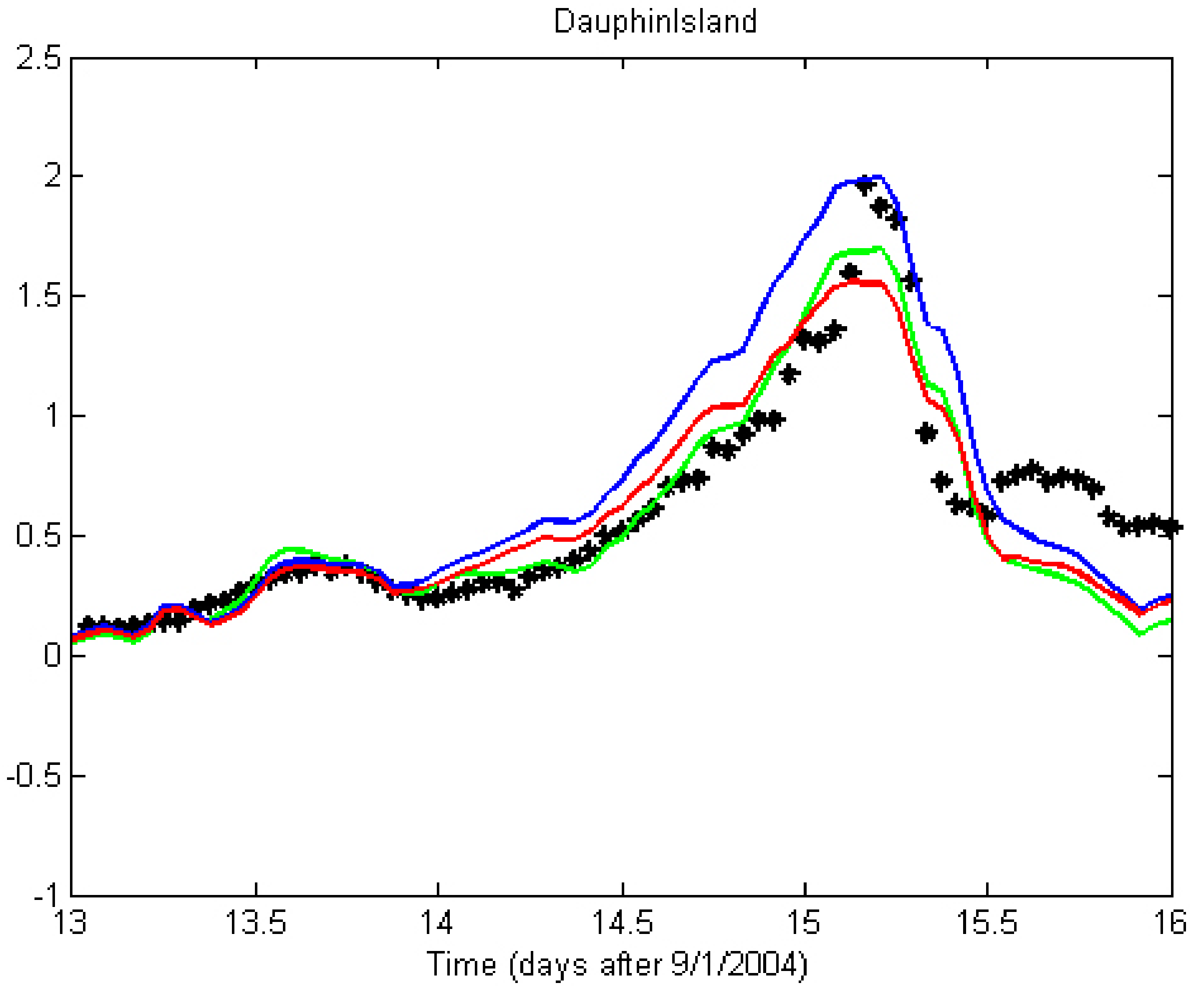

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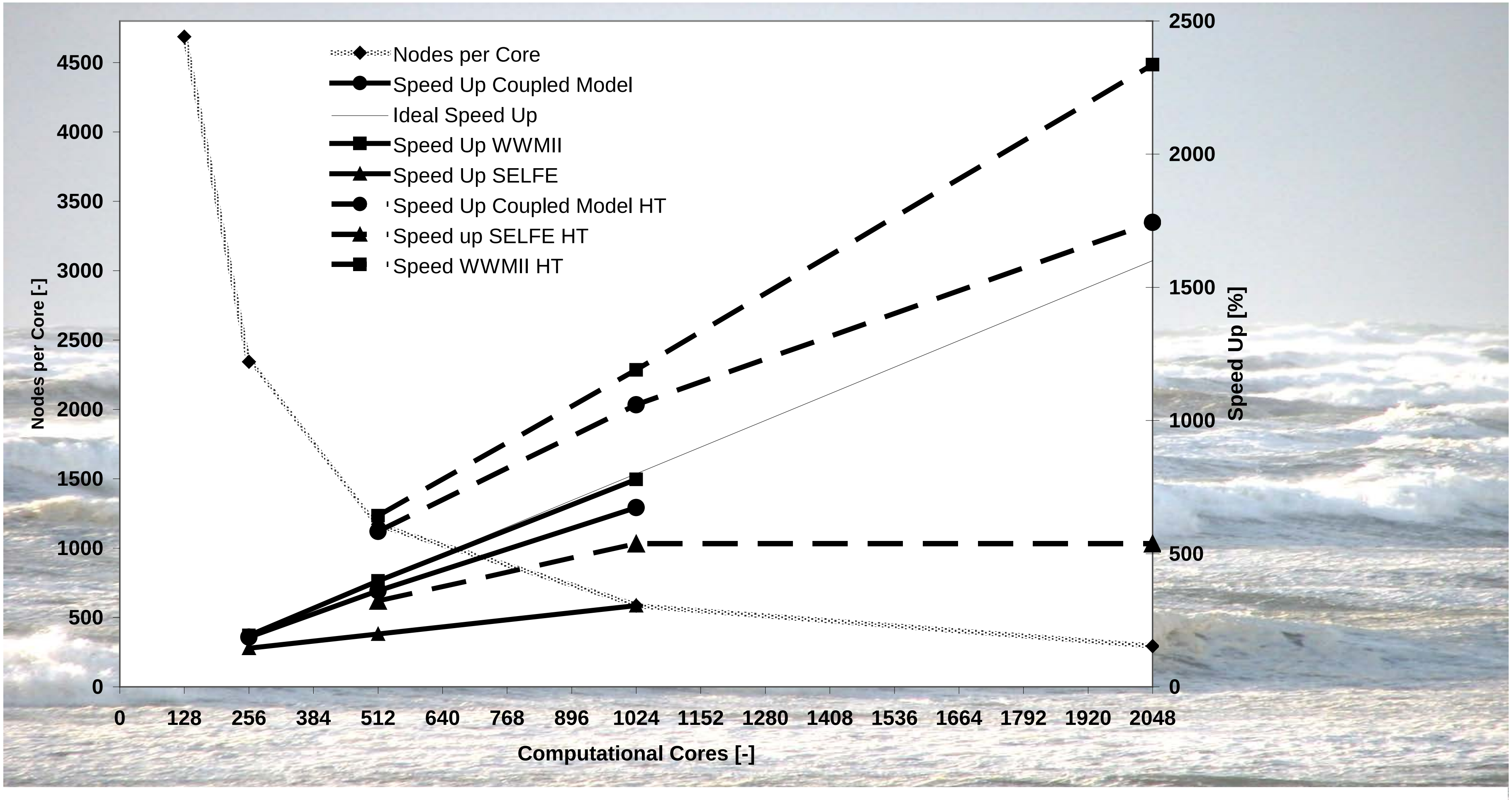

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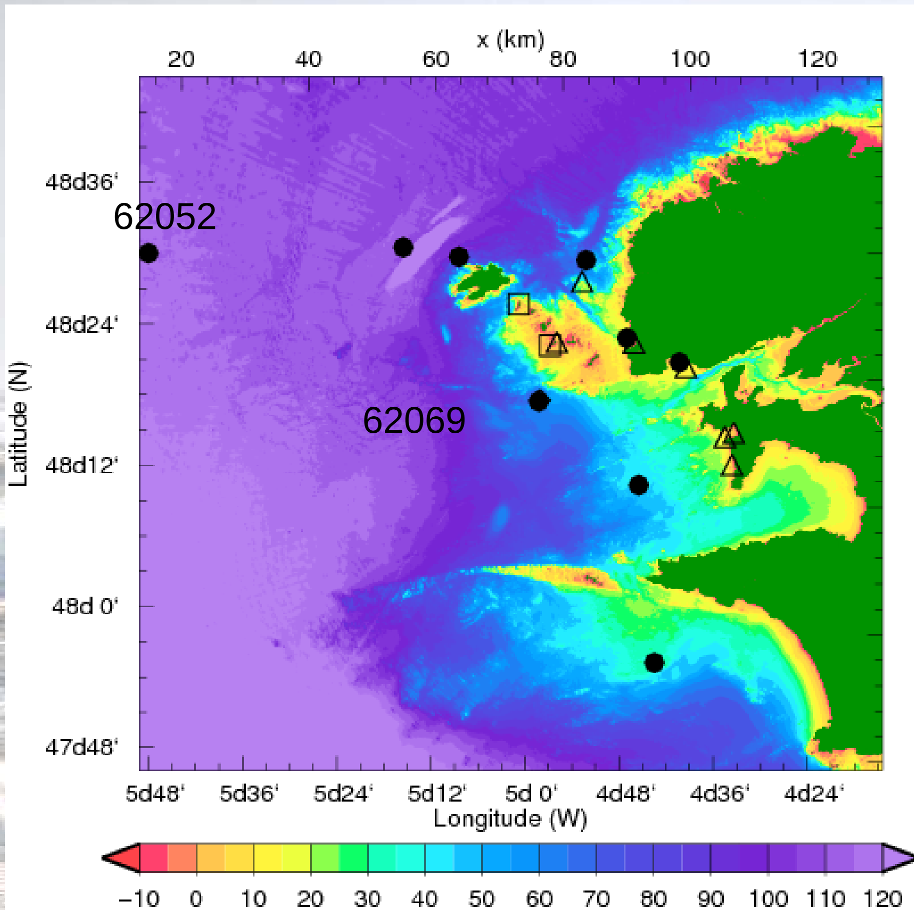
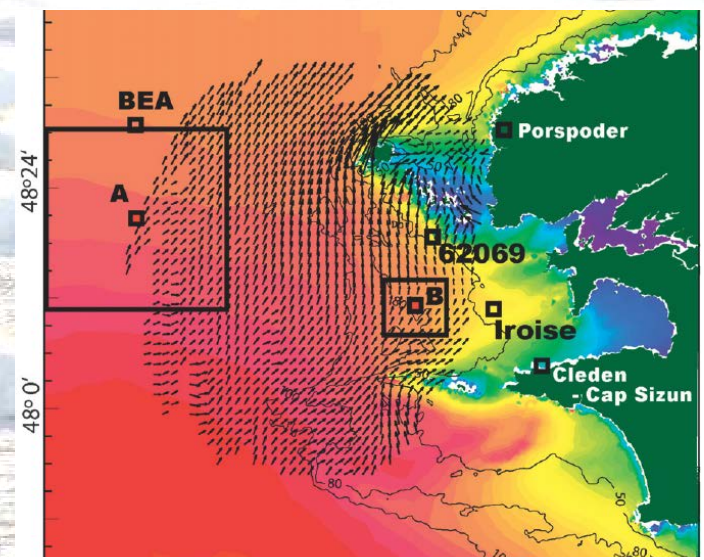
Applications of the RD-Schemes in WWM or WW3
West France - Brest





Ardhuin, F., Benis A-C, Roland, A., Filipot, J-F., Magne, R.,, Semi-empirical dissipation source functions for ocean waves: Part II, evaluation in conditions with strong currents., Journal of Physical Oceanography. J. Phys. Oceanogr, 2011, 40, 1917–1941. (submitted)

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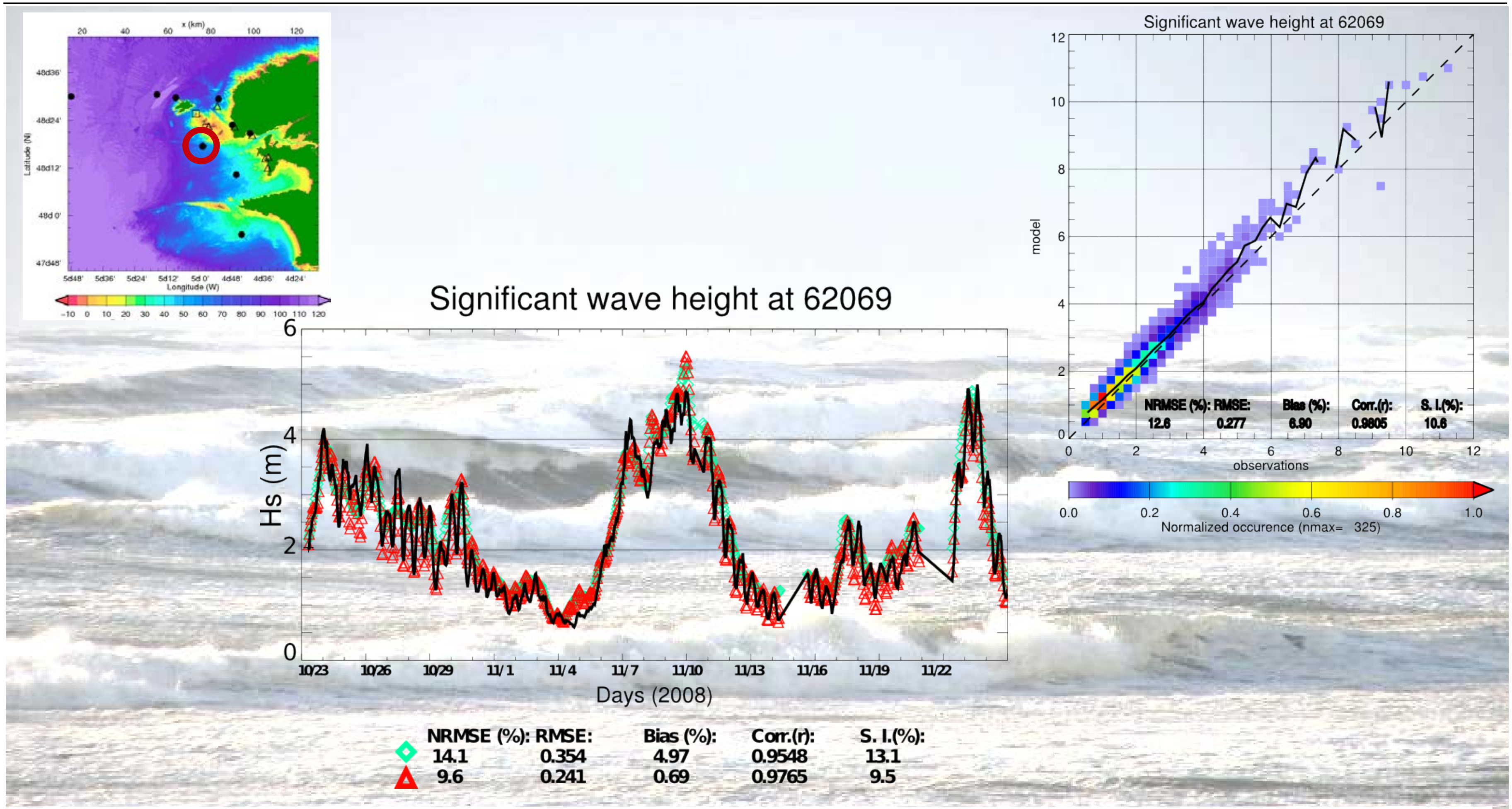

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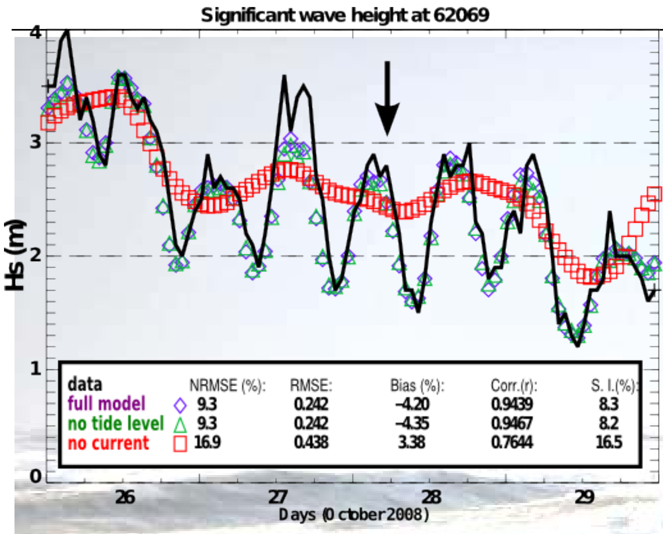

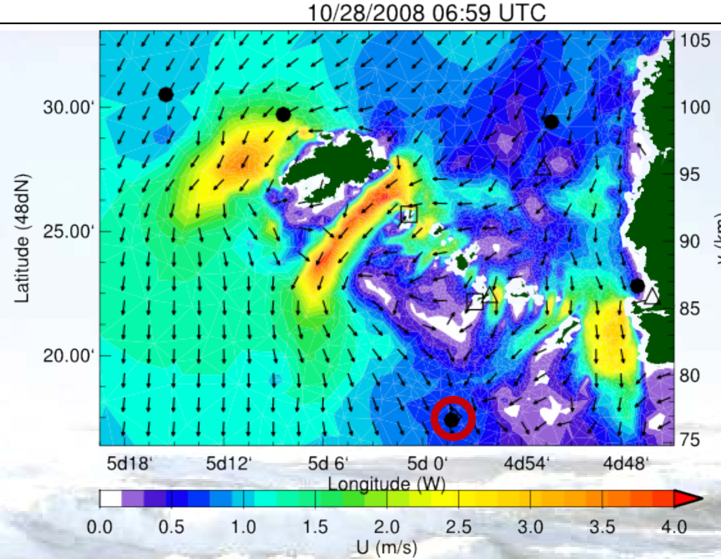

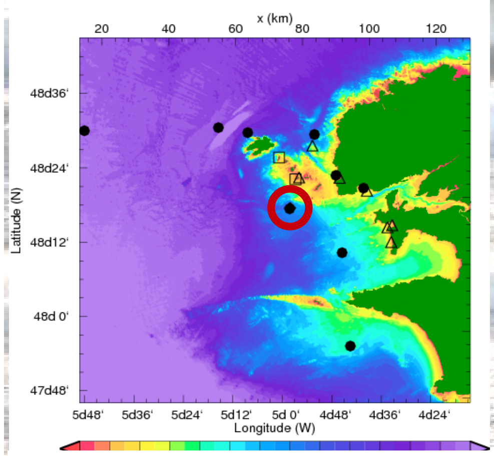

Case with steady offshore waves (Tp=10 s, Hs = 3 m) from the North-West

.... but very unsteady sea state due to strong changes in water levels!

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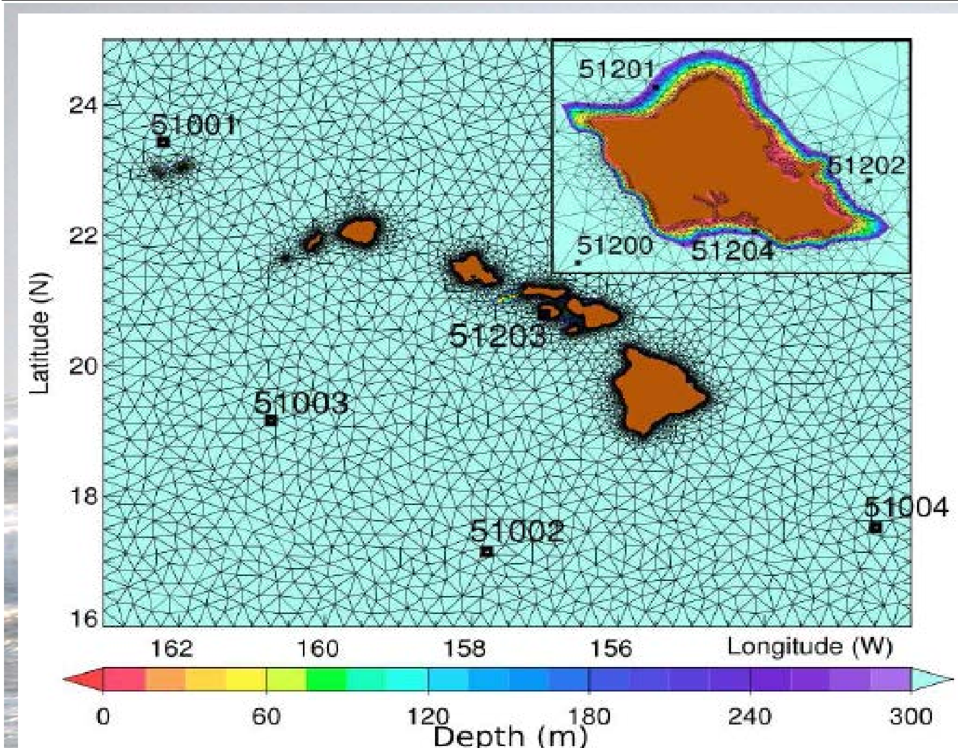
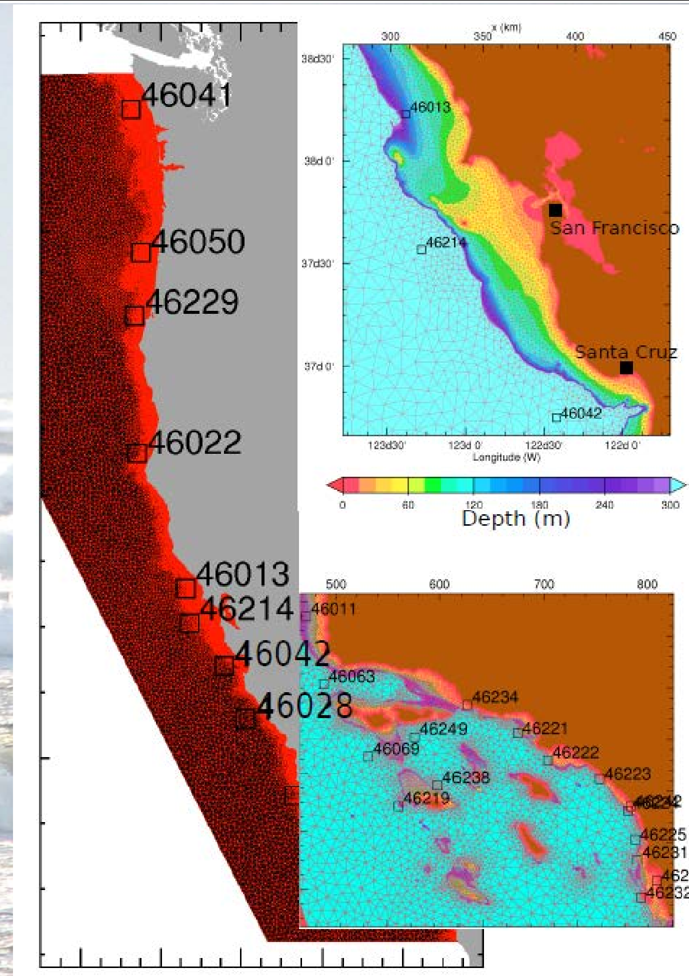
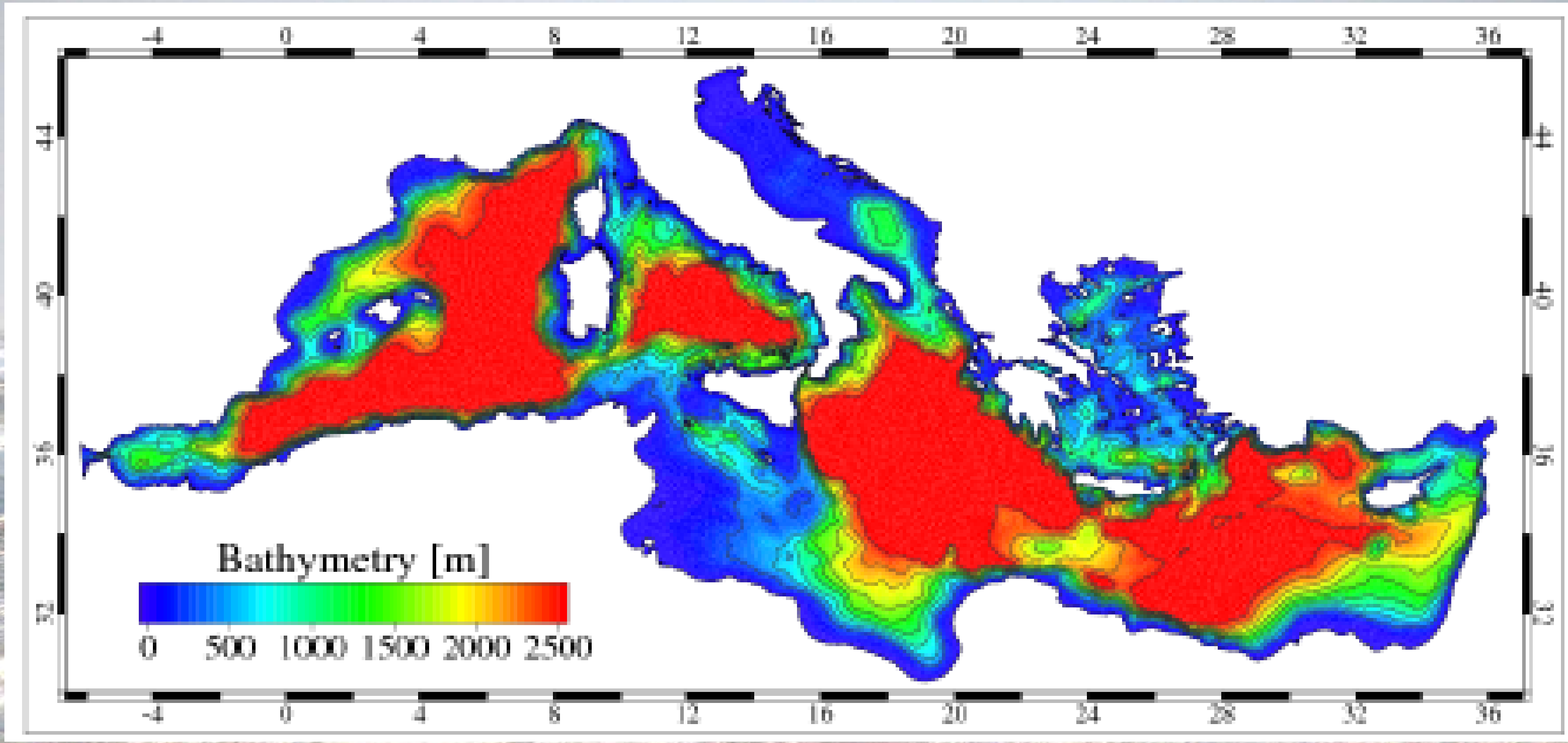
Applications of the RD-Schemes in WWM or WW3
Hawaii – U.S. West Coast - Coastal Reflection in WW3





Ardhuin, F., Roland, A., “Coastal Reflection in Spectral Wave Models”, submitted to JGR Oceans.

Kassandra – Operational Modeling of the Mediterranean SHYFEM-WWMII

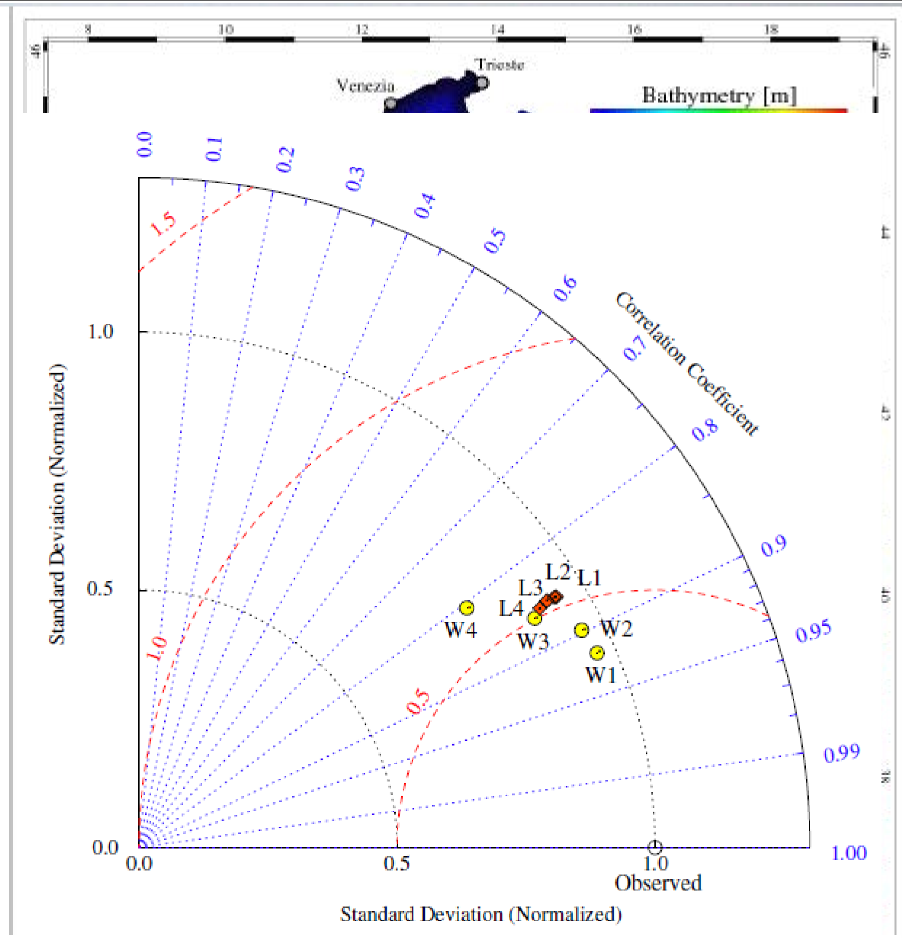


Ferrarin, C., Bajo, M., Roland, A., Umgiesser G., Tide-surge-wave modeling and forecasting in the Mediterranean Sea with focus on the Italian coast", submitted Ocean Modelling.

Applications of the RD-Schemes in WWM or WW3

Kassandra – Operational Modeling of the Mediterranean SHYFEM-WWMII

- **Kassandra**
- **High resolution atmospheric forcing.**
- **Fully coupled wave-current model in 2d.**
- **Operational since 2 years.**



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Challenges and dead ends when using unstructured meshes

- When the waves approach coastal regions, the spatial and temporal time scales in which the wave spectra changes become smaller and shorter. This needs to be accommodated in the spatial and temporal discretization of the region of interest as well.
- Unstructured mesh methods are with respect to this superior to structured grid methods but the price to pay in their development seems to be enormous, at least for me …
- The problems are:
  - Splitting Errors for methods that apply certain kind of splitting to the WAE.
  - Iterative one-step methods that may suffer by an ill conditioned matrix due to extremely large Eigen values.
  - For both problems the reason are stiff local contributions due to, sources and sinks or/and large contribution due to shifting in spectral space.
- Other problems are given by numerical diffusion/dispersion. We need a honest investigation of numerical effects in spectral wave models for multi-scale applications.

Numerical schemes

Operator Splitting Methods III (Fractional Step Method + Explicit Sources)

- **Operator Splitting Methods (OSM) e.g. WWIII or WWM**

  - **1st Step – Spectral part**

$$\frac{\partial N^{*}}{\partial t}+\frac{\partial}{\partial \theta}\left(c_{\theta} N\right)=0;\left[N^{*}{}_{(t=0)}=N_{0}\right] \text{ on } [0, \Delta t]$$

$$CFL_{\theta}=\left|\frac{c_{\theta} \Delta t_{\theta}}{\Delta \theta}\right|<1$$

$$\frac{\partial N^{**}}{\partial t}+\frac{\partial}{\partial \sigma}\left(c_{\sigma} N^{*}\right)=0;\left[N^{**}{}_{(t=0)}=N^{*}{}_{(t=\Delta t)}\right] \text{ on } [0, \Delta t]$$

$$CFL_{\sigma}=\left|\frac{c_{\sigma} \Delta t_{\sigma}}{\Delta \sigma}\right|<1$$

  - **2nd Step – Geographical space**

$$\frac{\partial N^{***}}{\partial t}+\frac{\partial}{\partial x}\left(c_{x} N^{**}\right)+\frac{\partial}{\partial y}\left(c_{y} N^{**}\right)=0 ;\left[N^{***}{}_{(t=0)}=N^{**}{}_{(t=\Delta t)}\right] \text{ on } [0, \Delta t]$$

$$CFL_{x}=\left|\frac{c_{x} \Delta t_{x}}{\Delta x}\right|<1$$

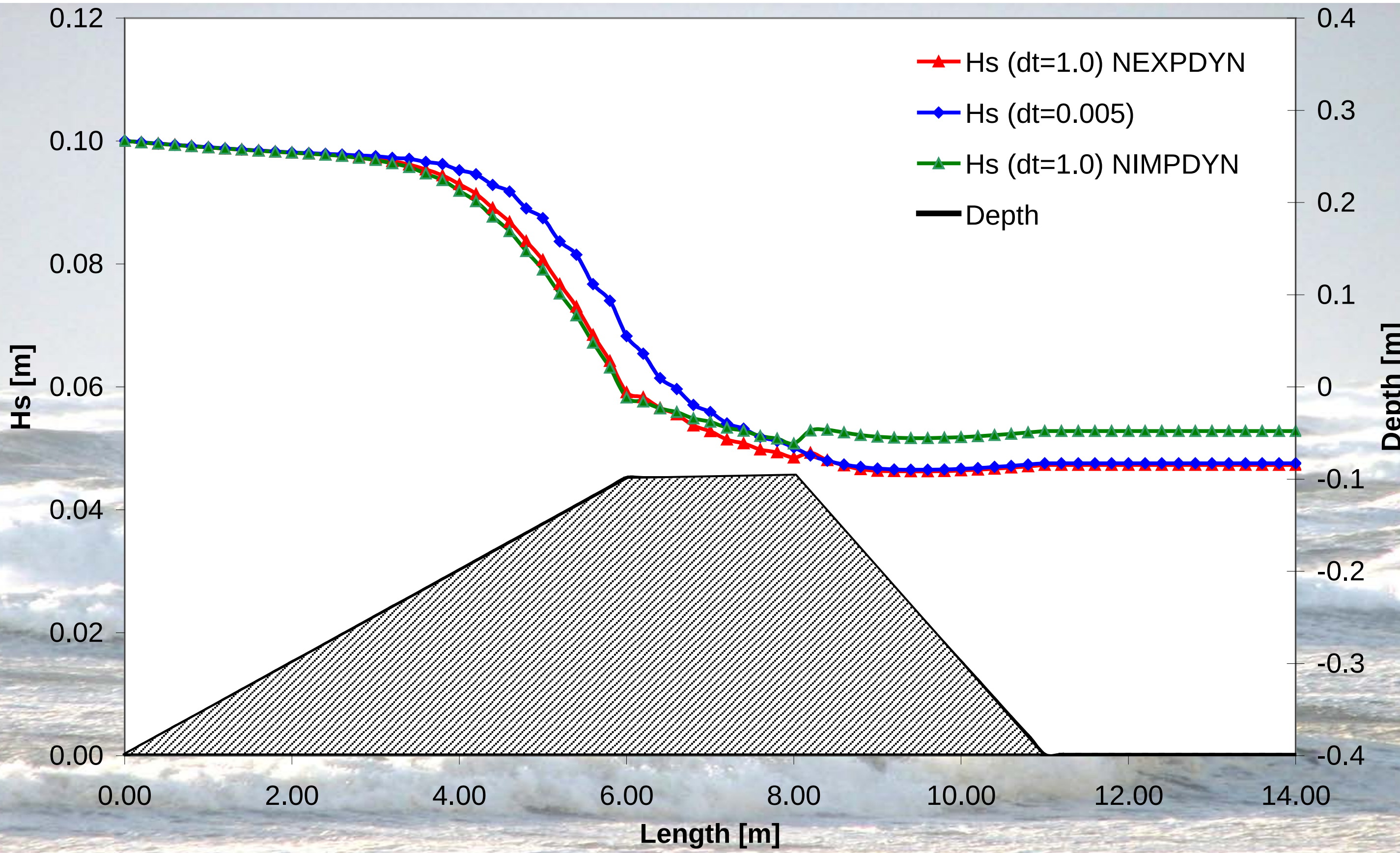
  - **3rd Step – Integration of the source terms**

$$\frac{\partial N^{****}}{\partial t}=S_{(N^{**}), tot};\left[N^{****}{}_{(t=0)}=N^{***}{}_{(t=\Delta t)}\right] \text{ on } [0, \Delta t]$$

Splitting Error between advection and strong local sources

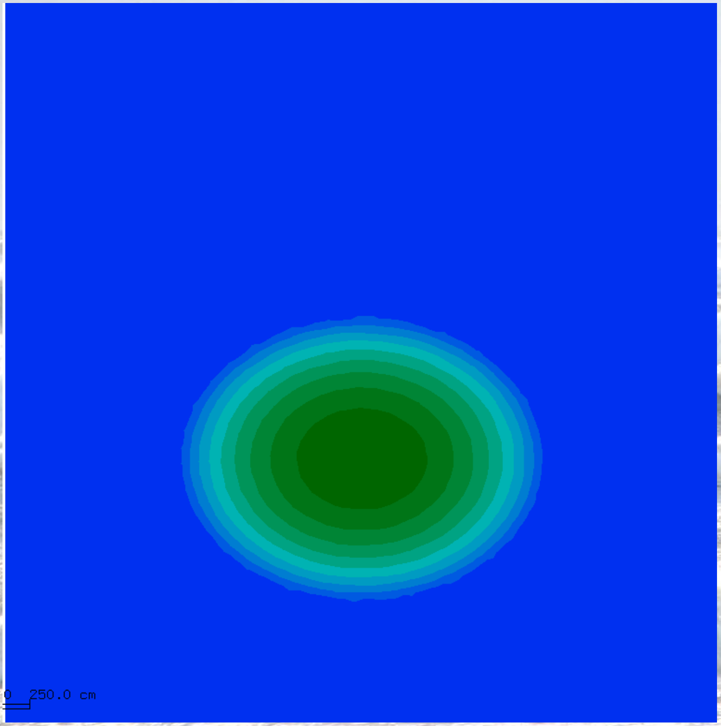
CFL_X=14

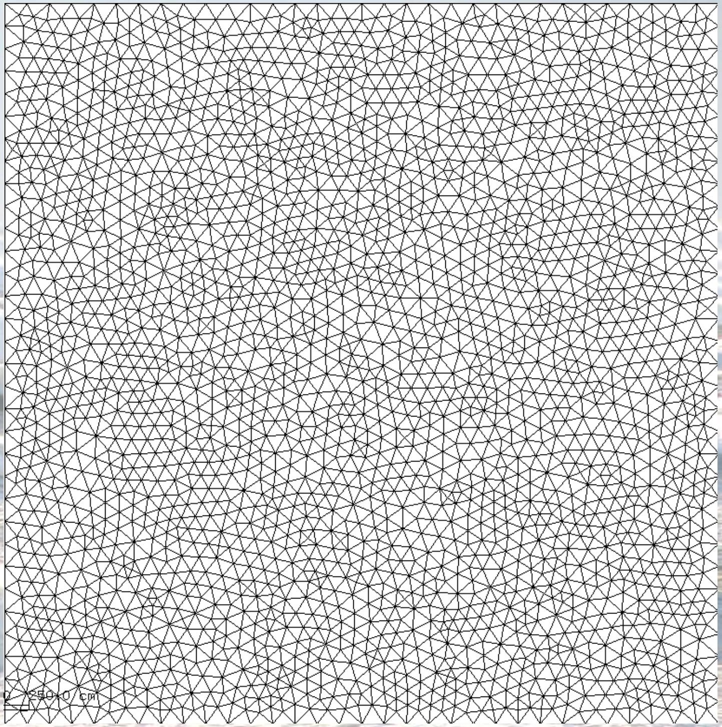
For Δt=1.0s



Significant wave height along a cross section for the unsplitted solution (blue) with Δt = 0.005 compared to the splitted solution using the explicit CRD-N scheme (red) and the implicit CRD-N1 scheme (green)

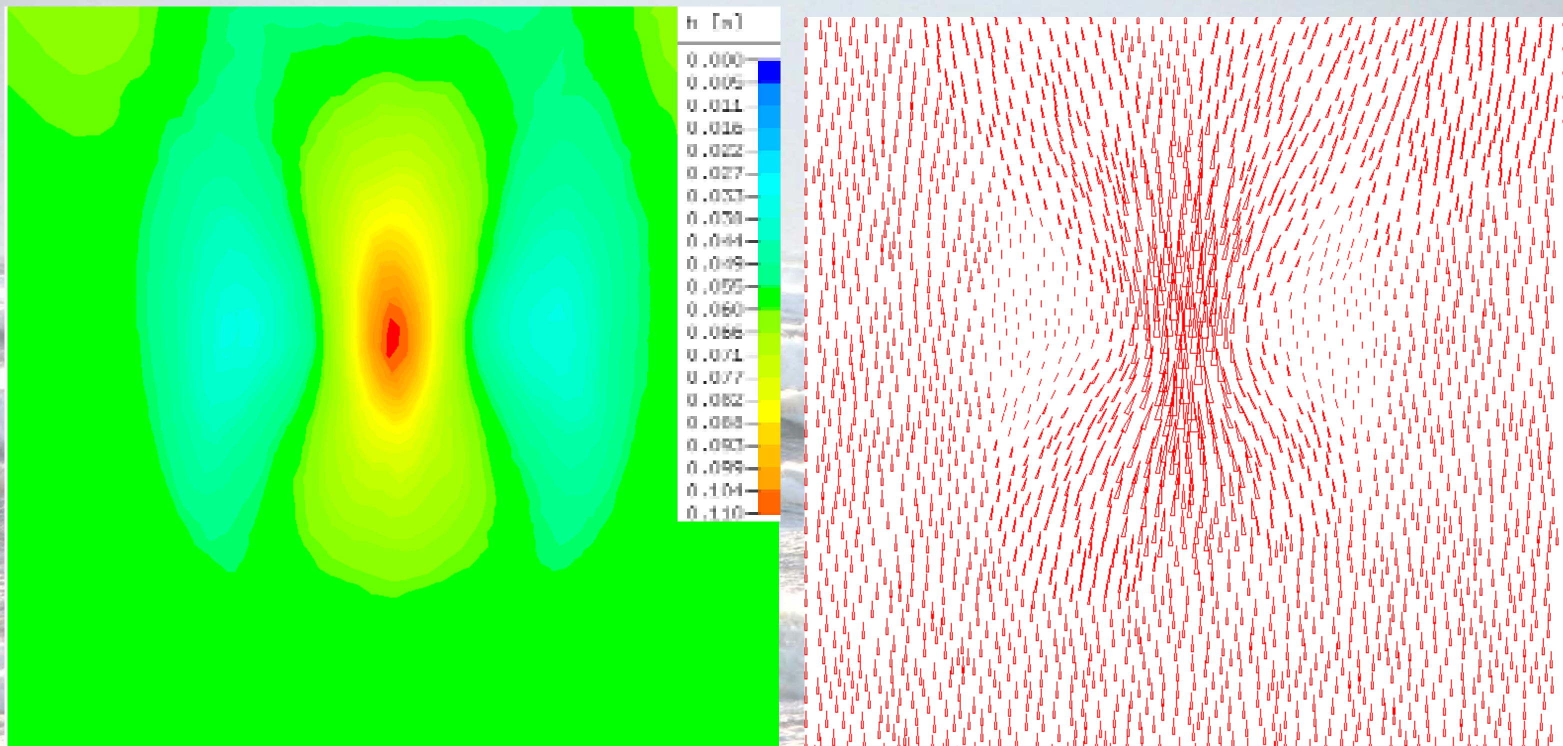
Splitting Error
Advection in Geographical and spectral space





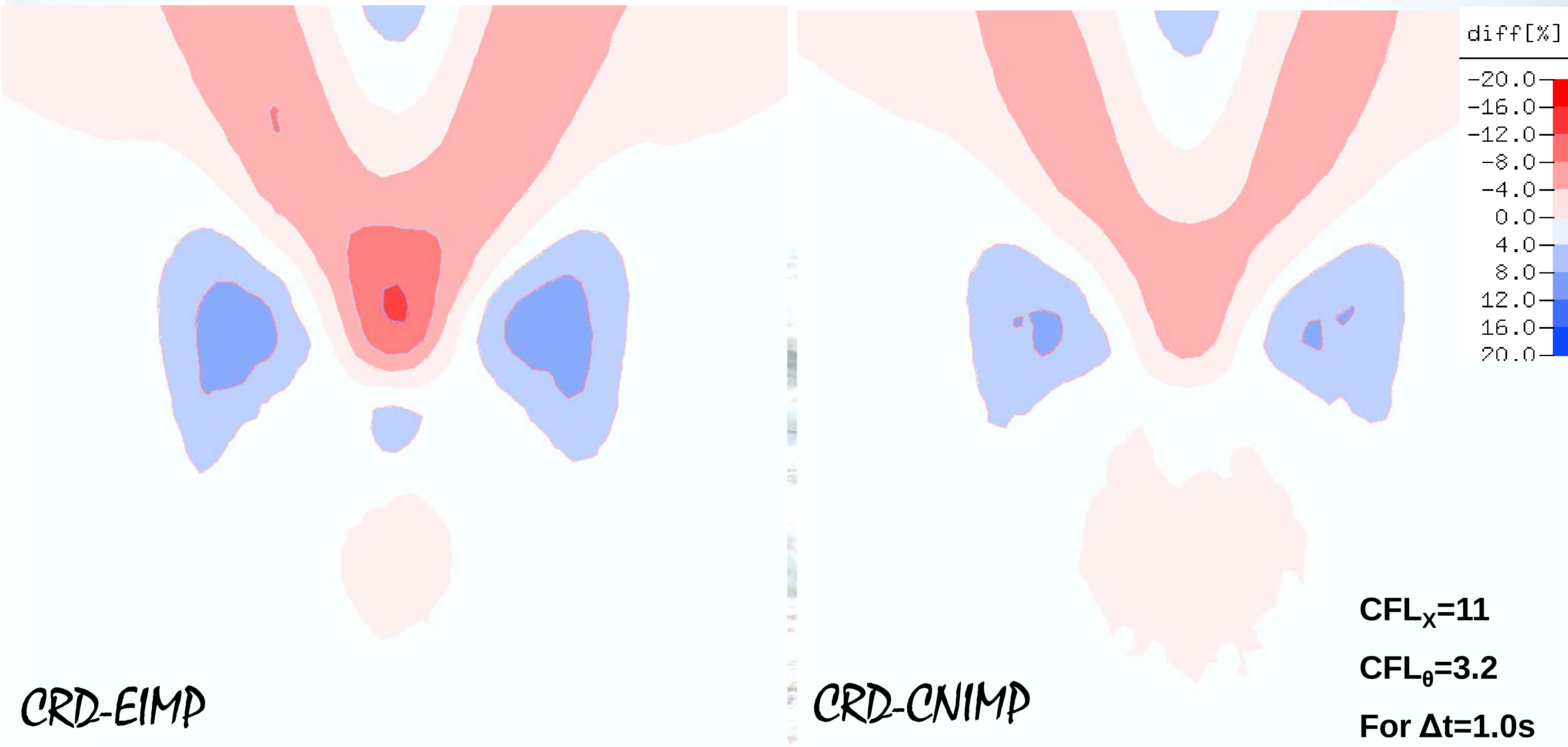
Bathymetry (left) and Computational mesh (right).

Splitting Error
Advection in Geographical and spectral space

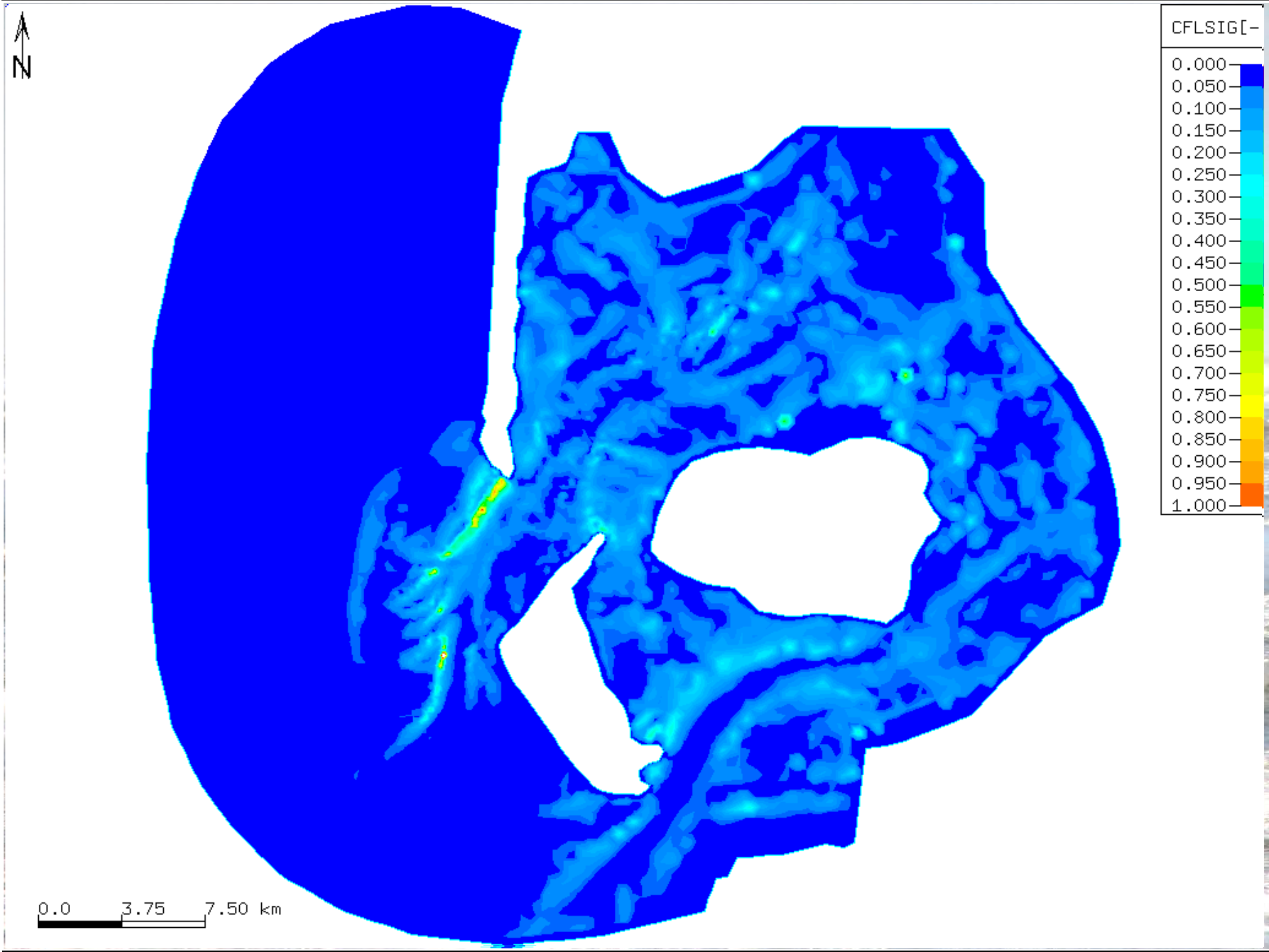


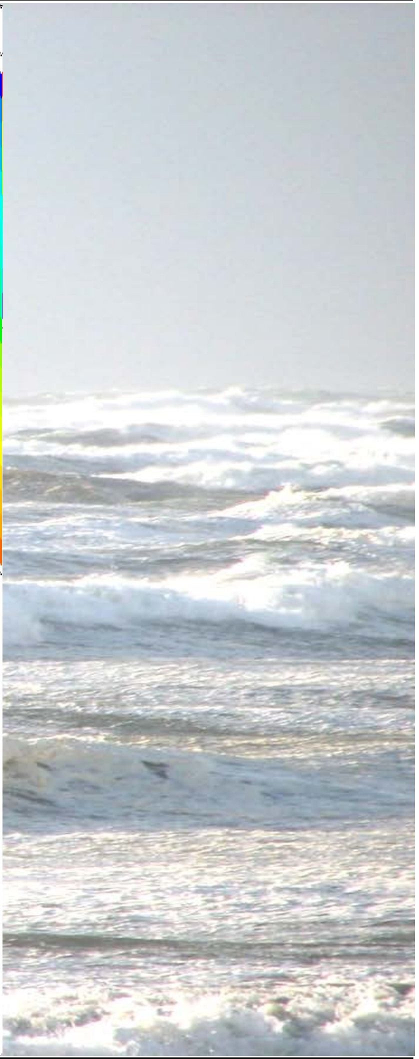
Wave height (left) and Average wave direction (right).

Splitting Error
Advection in Geographical and spectral space

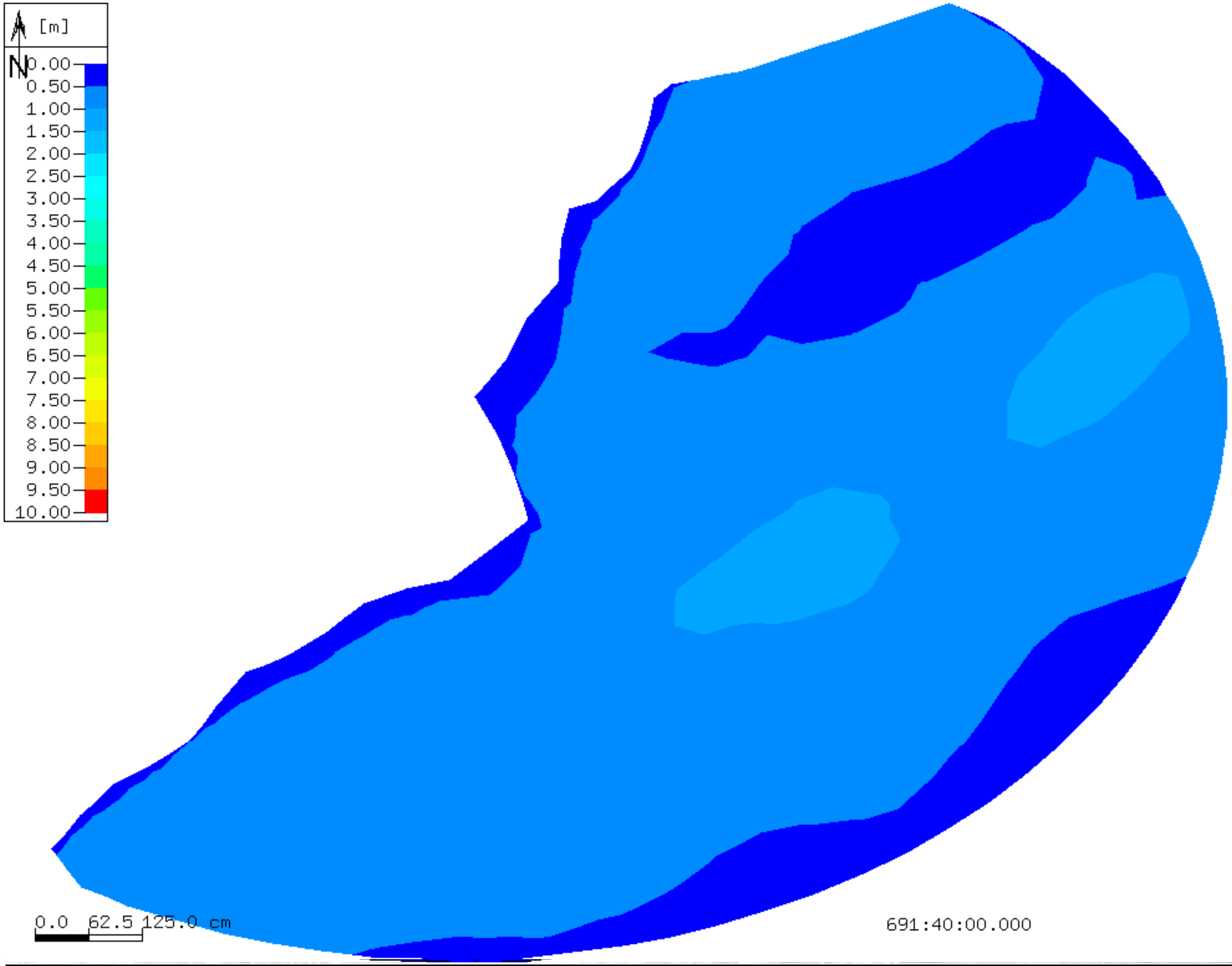


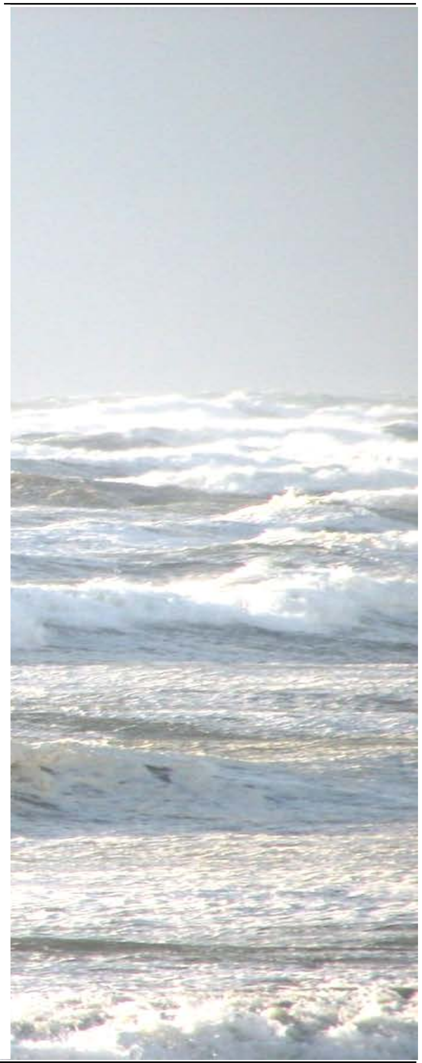
CFL numbers for instance in tidal basin’s



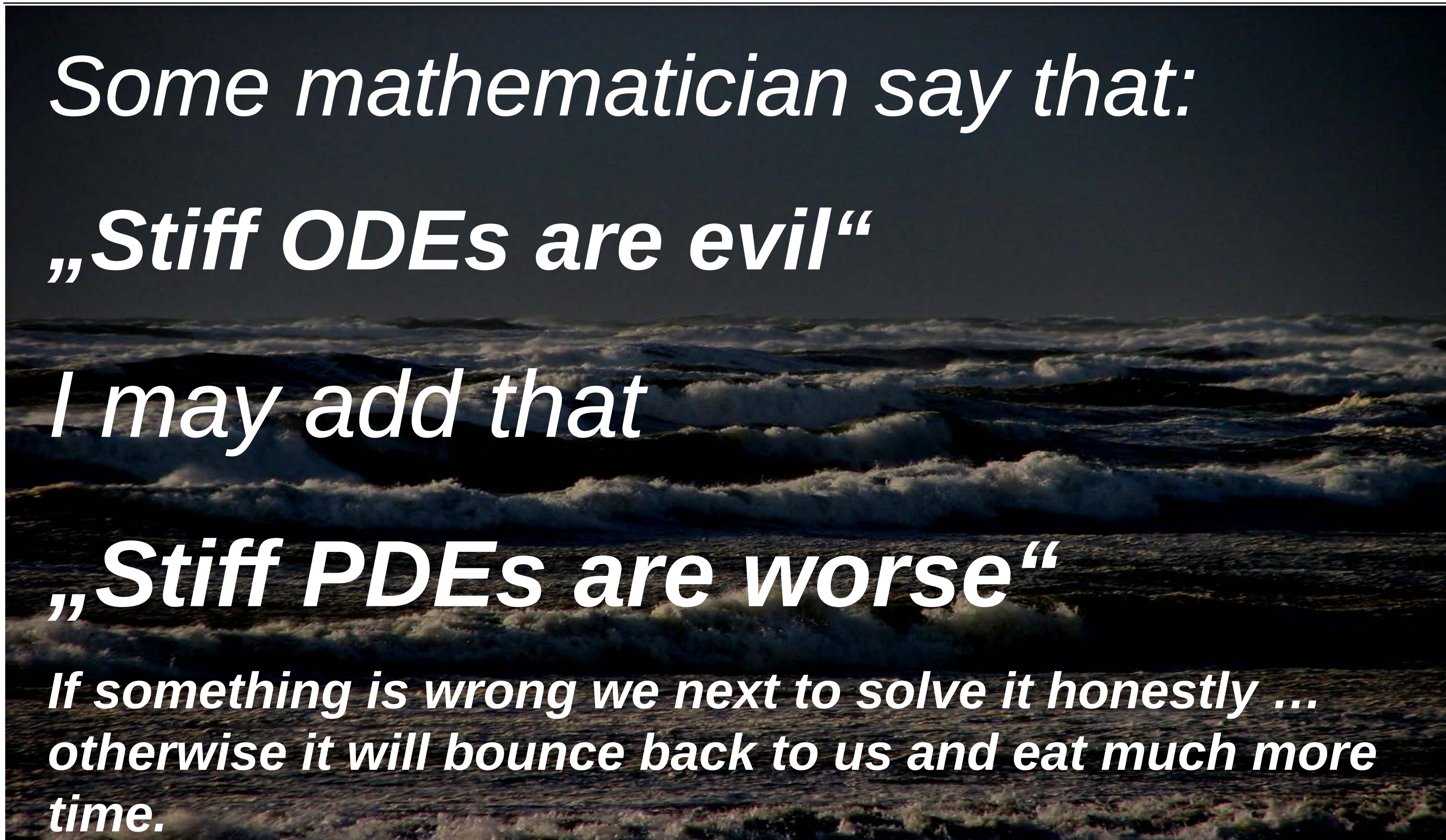


One-step methods have their problems too …





Building Blocks for a new Approach on unstructured Meshes for multidimensional stiff nonlinear PDE’s



Some mathematician say that:

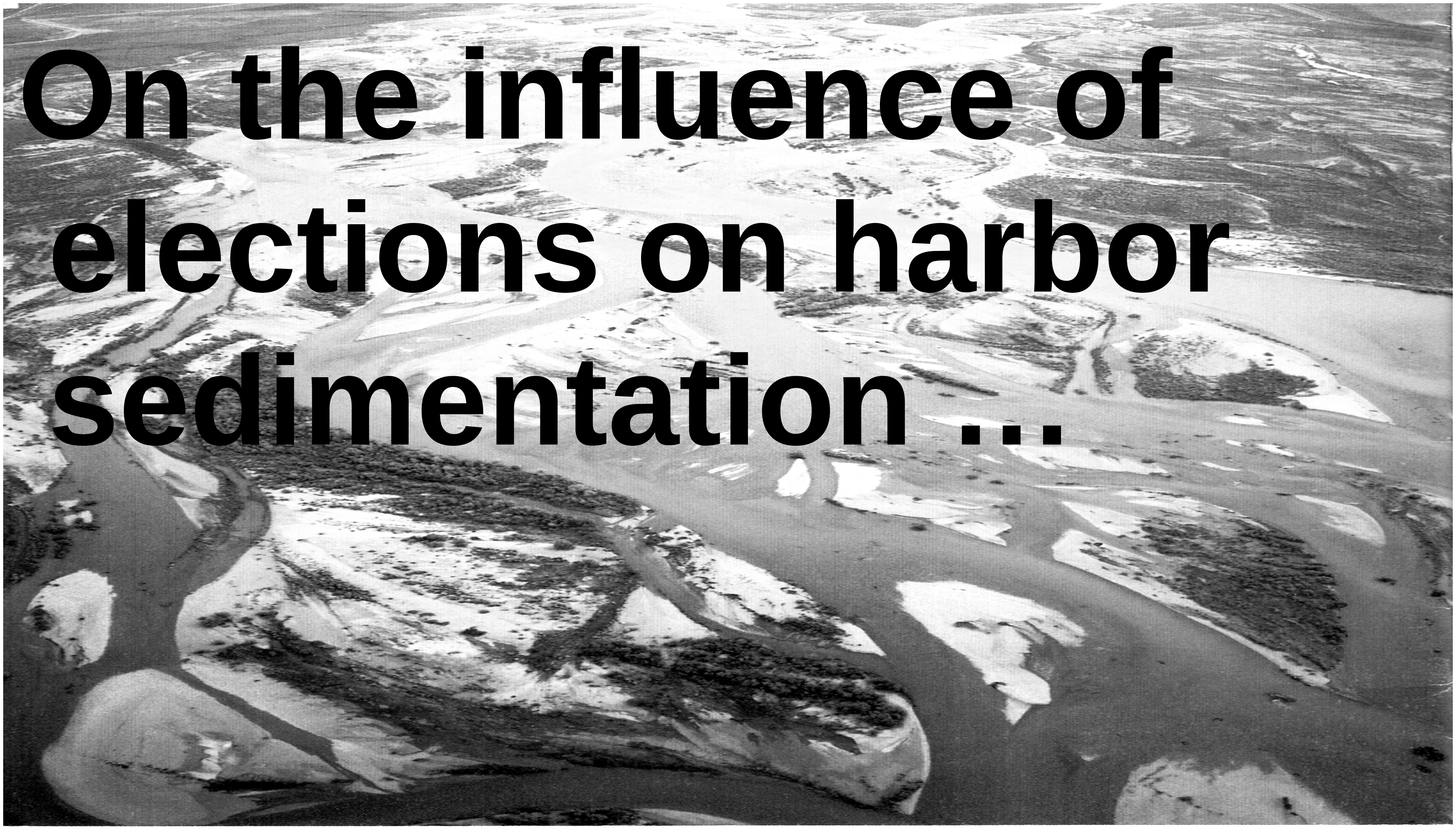
„Stiff ODEs are evil“

I may add that

„Stiff PDEs are worse“

If something is wrong we next to solve it honestly … otherwise it will bounce back to us and eat much more time.

Golden Beach - Taiwan



Golden Beach, Homei, Taiwan

Golden Beach, Homei, Taiwan

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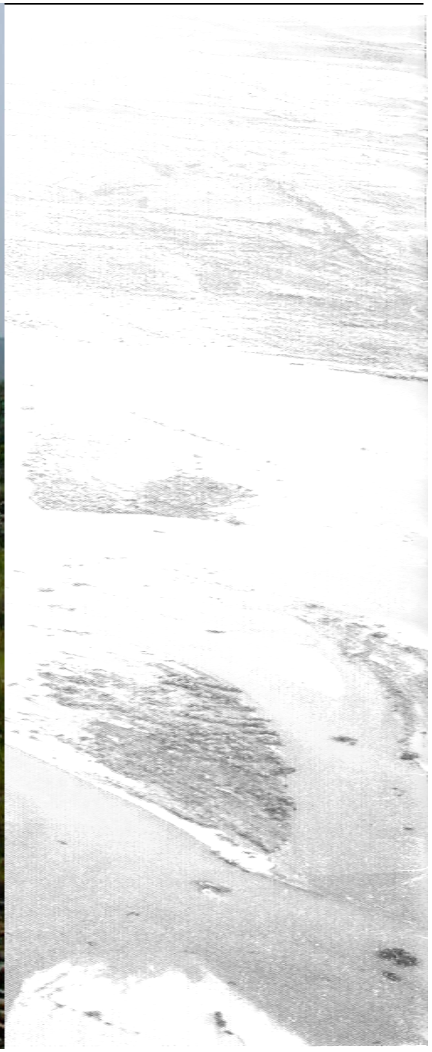

Golden Beach, Homei, Taiwan

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Golden Beach, Homei, Taiwan

Outlook

- Future Research
  - Development of a fully implicit WWM-III and WW-III
  - Morphodynamic modelling of the coastal zone
  - Long Term Morphodynamics
  - Wave-Current Interactions
  - Nonlinear propagation and EWAE

- Ongoing research
  - Modelling of storm surges, inundation and surface currents.
  - Optimization and validation of new numerical schemes.
  - High resolution modelling of Lagoons and tidal inlets with a focus on wave-current interactions in 3d.