Gravity waves in stable atmospheric boundary layers

Carmen J. Nappo

CJN Research Meteorology
Knoxville, Tennessee 37919, USA

Abstract

Gravity waves permeate the stable atmospheric planetary boundary layer, yet questions of origins of these disturbances remain unanswered. Before a parameterization of gravity waves can be effected, their sources and amplitudes must be known or stochastically predicted. The mechanisms of these generations must be resolvable in the numerical model. This precludes global waves generated outside the model domain, yet their actions affect the boundary layer as much as or even greater than locally generated waves. However, it is uncertain how to distinguish between global and local gravity waves. In this note, we examine two types of locally-generated waves and turbulence which could be parameterized in a numerical model.

1. Introduction

Before we can parameterize the effects of gravity waves on the atmospheric stable planetary boundary layer (PBL), the origins of these waves must be known or statistically estimated. These wave origins or statistics must be resolvable within the model and be functions of the model variables and parameters. We shall refer to these waves as local, and waves that originate outside of the model domain we shall call global. In the real PBL, both local and global disturbances exist simultaneously and generally independently. Examples of local wave mechanisms include shear instability, mountain waves, mountain lee waves and canopy waves. Examples of global waves include jet streaks, solitary waves, baroclinic inertia-gravity waves (e.g. geostrophic adjustment), thunderstorm lee waves, gust fronts, and mesoscale gravity waves often of unknown origin. Distinguishing between local and global waves based only on limited observations is difficult or even impossible. Yet, if a model parameterization is to be tested, the effects of global waves on PBL measurements must be either removed from the comparison data or explained.

Another possible difficulty, although seldom considered, is the dynamic stability of the wave. The linear theory is based on stable waves, i.e. waves with real wavenumbers and frequencies[1]. The linear theory assumes that the wave perturbations are small relative to mean or background values. Two possibilities related to this assumption exist: either wave amplitudes are too small to be observed although their possibility is predicted, or wave amplitudes are observed but too weak to have any effect on PBL turbulence. It would, perhaps, be prudent to consider all observed waves unstable, i.e. complex wavenumbers or frequencies. Chimonas (2002) suggests that maximum growth rates in PBL-generated waves is about 24 hours. Considering the often observed intermittency of gravity waves in the PBL, we might safely assume that these waves have constant or quasi-constant amplitudes. In this note, we explore the conditions for local gravity waves, and examine how these waves can affect PBL turbulence. Some suggestions for parameterizations are made.

[1] Note, when wavenumbers and frequencies are complex exponential growth of wave amplitude is possible. In these cases, linear theory breaks down.

2. Waves and turbulence

Considerable work has been done on the relation between gravity waves and turbulence in the stable PBL, and from these studies a reasonable understanding of wave-turbulence dynamics has been achieved. However, while some waves can be directly linked in some way with turbulence, is it not clear that all turbulence is directly linked with waves? It has been said, that all turbulence in a stably stratified flow is due to wave instability, and so the starting point of any analysis of flow instability usually beings with the Taylor-Goldstein equation. However, the sources of these wave-like motions leading to turbulence remain obscure. An analysis of wave instability consists of looking for the conditions where either wavenumbers or frequency or both become complex values. When this happens, wave amplitudes grow as $e^{m_i z}$ or $e^{\omega_i t}$, where m_i and ω_i the imaginary components of vertical wavenumber and wave frequency respectively.

3. Local gravity waves

In this section we will examine two sources of local gravity waves which potentially can lead to turbulence.

3.1. Mountain lee waves

Mountain lee waves are stationary waves trapped between the ground surface and some upper wave-reflecting level (see, for example, Smith, 1976). Because lee waves are resonant modes, theory predicts they will extend for long distances downwind of the mountain. However, Smith, et al (2002) observed that under conditions for lee waves downwind of Mont Blanc, the waves decayed quickly in amplitude or did not appear at all. They proposed that a strongly stable PBL can extract momentum and energy from the lee wave, thus reducing wave amplitude.

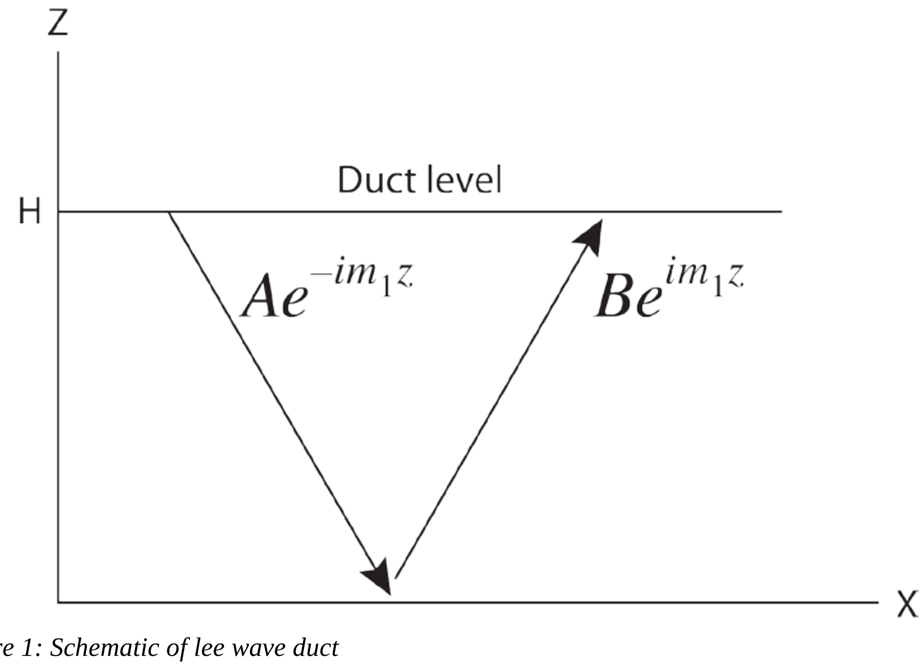

Figure 1: Schematic of lee wave duct

Figure 1 is a schematic diagram of a lee wave duct. The wave with amplitude A is reflected downward at the duct height H. The wave with amplitude B is reflected upward at the ground surface. For perfect reflection at the ground surface we require

$$A + B = 0 \tag{1}$$

Smith etal (2002) proposed partial reflection at the ground surface such that:

$$A + qB = 0\,, \tag{1}$$

where q is a reflection coefficient such that for q = 1 there is complete reflection and for q = 0 there is complete absorption of the wave. Figure 2 plots the decay of lee wave vertical velocity, w, as a function for downwind distance for various values of reflection coefficient q. When $|w|$ is plotted on a log scale, the decay is a linear function of downwind distance x.

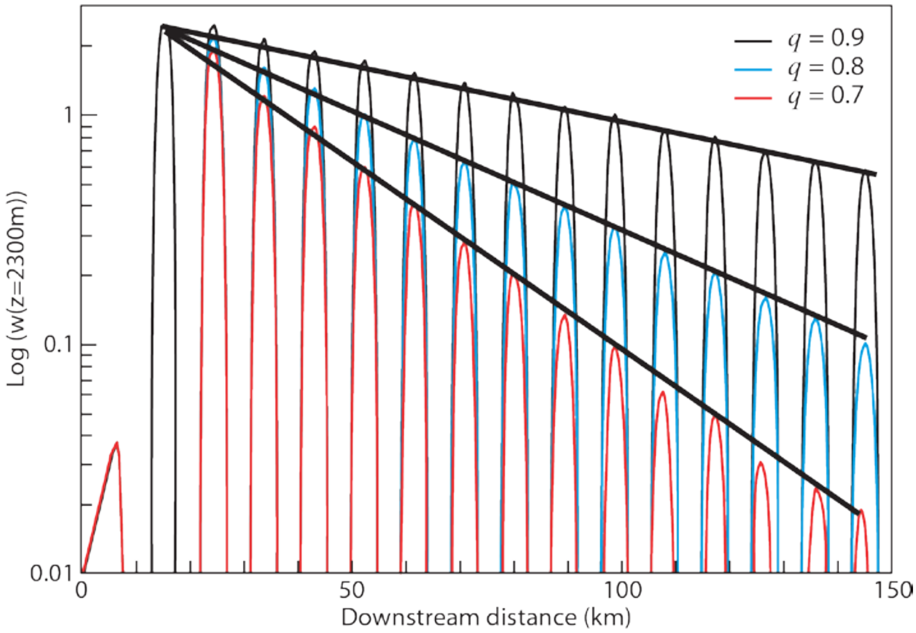

Figure 2: Magnitude of lee side vertical velocity as functions of downwind distance for various values of reflection coefficient q. (Jiang, et al, 2006)

The downwind decay of w results in a downwind decay of horizontal wave stress, $\tau = \overline{\rho_0 wu}$, which acts as a downward momentum flux in the boundary layer. Jiang et al (2006) develop the expression for this stress

$$\tau(x,o) = 0.25\rho_0 w_0^2 \left(1 - e^{-2\alpha x}\right)\frac{H}{x} \tag{3}$$

where w_0 is the maximum vertical velocity at some point x = 0 (which can be taken as the crest of the mountain), and α is a decay coefficient which is a function of q and the Scorer parameters below and above H. As sample evaluation of (3) on the PBL scale, we use the scheme in Fig. 3 which shows a schematic diagram of two-dimensional flow over a Gaussian-shaped ridge. In each layer wind speed and stratification are constant. The ridge is on the boundary layer scale with maximum height of

100 m and base width of about 4.4 km. Figure 4 plots the surface wave stress due to lee waves for various values of *q*. Unlike the vertical wave stress which is constant in the absence of wave dissipation, the lee wave stress can have a horizontal divergence and thus act as a drag on the PBL flow.

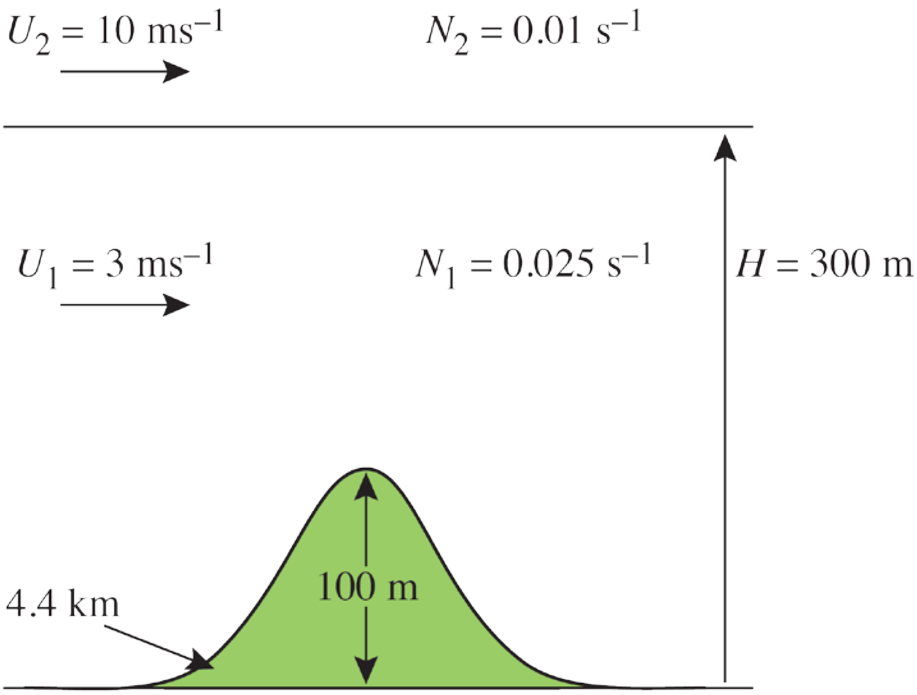

Figure 3: Two-layer flow over a two-dimensional Gaussian ridge

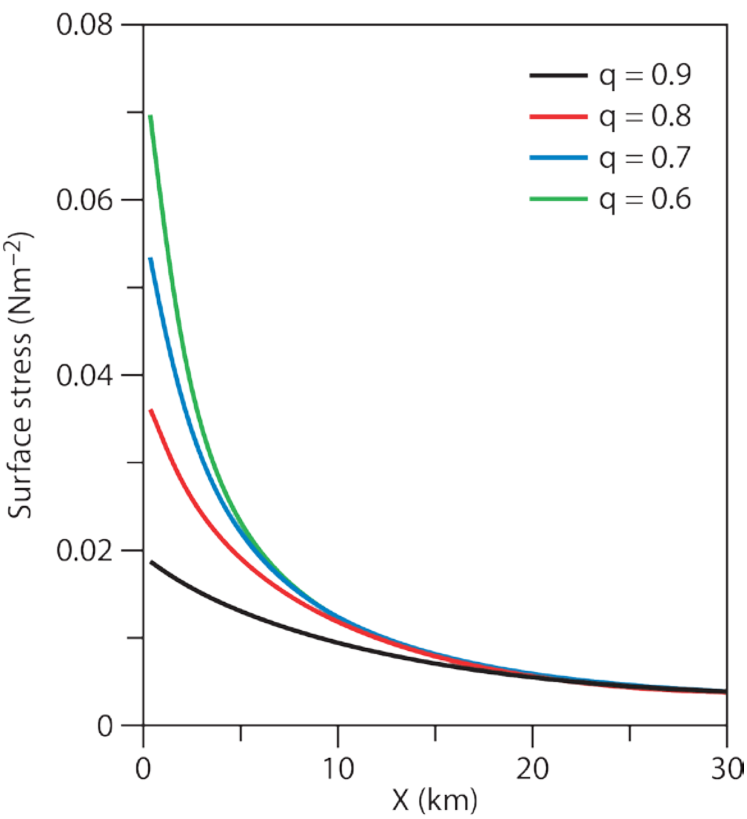

Figure 4: Lee wave surface stress as functions of reflection parameter q.

3.2. Unstable wave modes in the stable PBL

Chimonas (2006) described a set of gravity modes peculiar to the night-time PBL. His model boundary layer consists of a stable surface layer of depth h, a neutral residual layer with depth $h < z < H$, and a stable free troposphere extending from $z > H$. An analytical expression is used to specify the speed of the wind in the boundary layer, and for $x > H$ the wind speed is constant. The Richardson number is greater than 1 within the surface layer. The Taylor-Goldstein equation in the surface layer is

$$\frac{d^2w}{dt^2}+\left[\frac{N^2}{(c-U)^2}+\frac{1}{(c-U)}\frac{d^2U}{dz^2}-k^2\right]w=0 \tag{4}$$

Solutions of (4) that satisfy the boundary conditions $w(0) = 0$ and $w(\infty)\to 0$ for particular eigenvalues (c, k) represent horizontally propagating modes. In the free troposphere $z > H$, (4) has the form

$$\frac{d^2w}{dt^2}+\left[\frac{N_{upper}^2}{(c-U_{upper})^2}-k^2\right]w=0 \tag{5}$$

Equation (5) is applied at the $z = H$. The general solution to (4) is

$$w(z)=A\exp\left\{\left[k^2-\frac{N_{upper}^2}{(c-U_{upper})^2}\right]^{1/2}z\right\}+B\exp\left\{-\left[k^2-\frac{N_{upper}^2}{(c-U_{upper})^2}\right]^{1/2}z\right\} \tag{6}$$

The upper boundary condition, i.e. $w(z)\to 0$ as $z\to\infty$ requires that

$$\left[k^2-\frac{N_{upper}^2}{(c-U_{upper})^2}\right]^{1/2} \tag{7}$$

has a real part. If the real part of (7) is positive, then the mode is the solution with $A = 0$. If the real part of (7) is negative, then the mode is the solution with $B = 0$. Details of the calculations are given in Chimonas (2002) where typical boundary layer values are used for wind speed $U(z)$ and $N(z)$, $h = 100$ m and H=500 m. Neutral and unstable modes are found which all develop in the neutral residual layer. Figure 5 shows the vertical velocity profile for the fundamental neutral-stable mode with wavelength 317 m and phase speed 3.4 ms^{-1}. We see that for the neutral-stable case, all the wave energy is confined to the boundary layer. Figure 6 shows the vertical velocity profile for the first unstable mode with wavelength 940 m and phase speed 3.8 ms^{-1}. In the case of unstable modes there is leakage of wave energy out of the PBL. Unstable modes grow as $e^{c_i kt}$, where c_i is the imaginary part of the complex phase velocity. For the case shown in Fig. 6, the growth rate is $c_i k \approx 25\times10^{-3}$ s. This mode takes about 24 hours to grow by a factor of $e \approx 2.7$. Thus, Chimonas (2002), remarks that for all practical purposes these unstable boundary layer modes can be considered stable ducted waves.

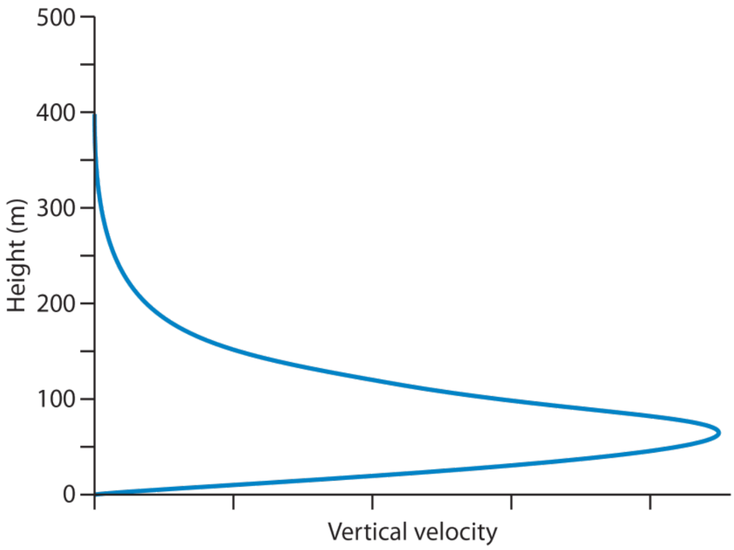

Figure 5: Vertical velocity profile for the neutral-stable fundamental mode. (Chimonas, 2002)

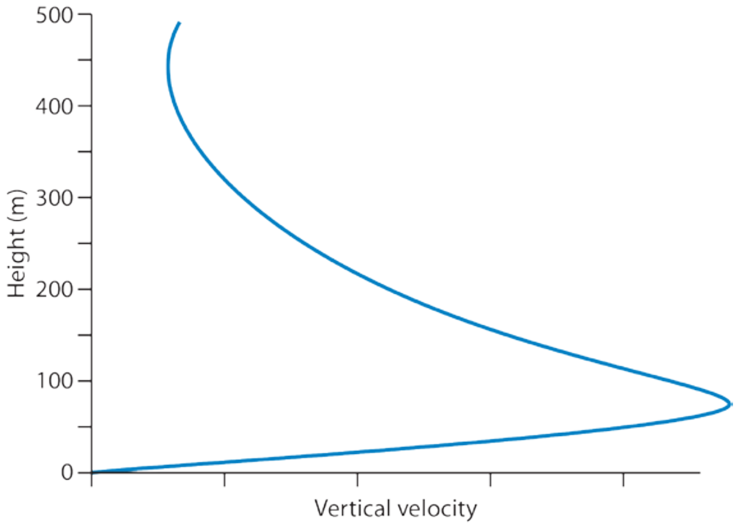

Figure 6: Vertical velocity profile for first unstable mode. (Chimonas, 2002)

In the real PBL, any horizontal component of the wind vector can generate modes if the residual layer if near neutral. Figure 7 shows the domain of phase velocities that can lead to ducted waves. The heavy line is the hodograph extending from the ground surface to the top of the PBL. It is this characteristic which makes these PBL modes seemingly ubiquitous in the night-time PBL

4. Discussion

We have described two mechanisms that can generate gravity waves in the stable PBL. Although the physics of these mechanisms is straightforward, incorporating them in a mesoscale numerical model may not be easy. The mountain lee wave mechanism has the greater potential for applications because the magnitudes of the wave stresses can be evaluated. Ducted modes can increase horizontal wind shear, and this can lead to turbulence through modulation of the local Richardson number. Because the PBL modes can propagate in many directions depending on the values their presence may be an integral part of the PBL flow. Because of their omnipresence, it may be possible to use a stochastic approach for wave amplitudes. We also note that a characteristic of gravity waves in the PBL is their short durations. Considering the vast number of eigenvalues that might differ by very small amounts, changes in wind speed, wind direction, boundary-layer height, and stratification can destroy a duct. Thus a total stochastic parameterization of PBL waves based on climatologies of PBL modes and related to background conditions would seem to be most useful. However, in developing climatology of PBL waves, one must distinguish between free tropospheric modes and PBL modes.

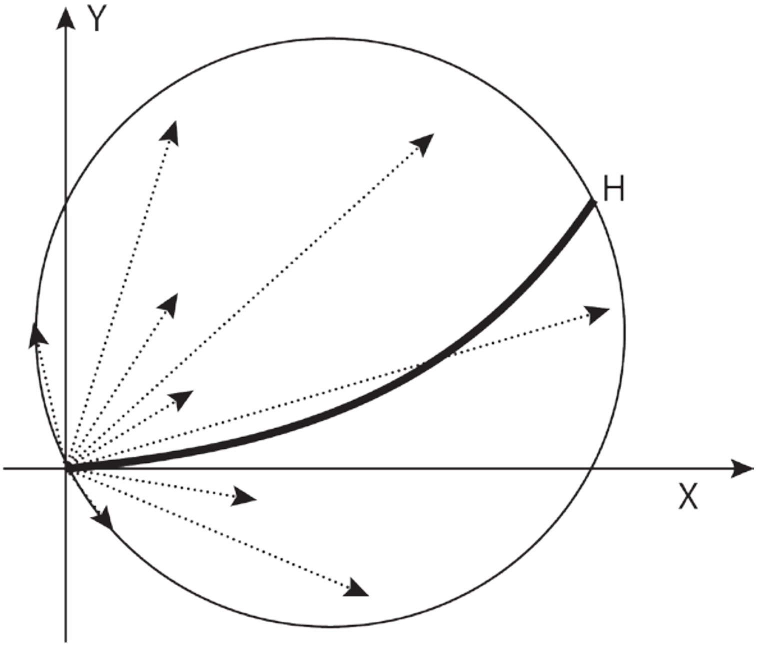

Figure 7: Circular domain of phase velocities that can lead to ducted modes (Chimonas, 2002)

5. Conclusions

We have described two mechanisms which can evaluate model wave structures originating in the stable PBL. One is based on mountain lee waves scaled to the PBL, the other is based on the generation of ducted modes in the PBL. These mechanisms are possible sources of intermittent turbulence, and can lend themselves to parameterization in numerical models.

References

Chimonas, 2002: On internal gravity waves associated with the stable boundary layer. *Boundary-Layer Meteorology,* **102**, 139-155.

Jiang, Q., J. D. Doyle, and R. B. Smith, 2005: Blocking, descent and gravity waves: Observations and modelling of a MAP northerly föhn event. *Quart. J. Roy. Metero. Soc*., **131**, 675-701.

Smith, R. B., 1976: The generation of lee waves by the Blue Ridge. *J. Atmos. Sci*., **33**, 507-519.

Smith, R. B., S. Skubis, J. D. Doyal, A. S. Broad, C. Kiemle, and H. Volkert, 2002: Mountain waves over Mont Blanc: Influence of a stagnant boundary layer. *J. Atmos. Sci*., **59**, 2073-2092.