

# Effect of ocean waves on ocean circulation

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## ABSTRACT

Ocean waves play an important role in processes that govern the fluxes across the air-sea interface and in the upper-ocean mixing. Equations for current and heat are presented that include effects of ocean waves on the evolution of the properties of the upper ocean circulation and heat budget. The turbulent transport is modelled by means of the level- $2\frac{1}{2}$  Mellor-Yamada scheme (Mellor and Yamada (1982)), which includes an equation for the production and destruction of Turbulent Kinetic Energy (TKE). The TKE equation in this work includes production due to wave breaking, production due to wave-induced turbulence and/or Langmuir turbulence, effects of buoyancy and turbulent dissipation.

As a first test, the model is applied to the simulation of the daily cycle in Sea Surface Temperature (SST) at one location in the Arabian sea for the period of October 1994 until October 1995. For this location, the layer where the turbulent mixing occurs, sometimes called the Turbocline, is only a few metres thick and fairly thin layers are needed to give a proper representation of the diurnal cycle. The dominant processes that control the diurnal cycle turn out to be buoyancy production and turbulent production by wave breaking, while in the deeper layers of the ocean the Stokes-Coriolis force plays an important role.

## 1 Introduction

Apart from the traditional benefits of sea state forecasting (e.g. for shipping, fisheries, offshore operations and coastal protection) it is now known that knowledge of the sea state is also important for a more accurate description of air-sea interaction, e.g. the sea state affects

- the momentum transfer,
- the heat transfer and
- the ocean surface albedo.

Here, I will briefly study sea state effects on the upper ocean dynamics and upper ocean mixing. Starting point is the Mellor-Yamada scheme where the turbulent velocity is determined from the **turbulent kinetic energy equation**. The turbulent velocity is then used to determine the eddy viscosities in the equations for momentum and heat. The turbulent kinetic energy equation describes the balance between the production of turbulent kinetic energy by work against the **shear** in the current and the Stokes drift (produces wave-induced and Langmuir turbulence), production by gravity **wave dissipation**, **buoyancy** and **dissipation of turbulence**. Note that momentum transport is also directly affected by the waves through the so-called **Stokes-Coriolis** force.

The mixed layer model has been run for a one year period from the 16th of October 1994 for a location in the Arabian Sea where extensive observations of temperature profile, current profile, solar insolation, wind speed, etc. were collected during the Arabian Sea mixed layer dynamics Experiment (ASE)

(Baumgartner et al. (1997); Weller et al. (2002)). Verification of modelled temperature profiles against these observations shows that dissipating waves and the Stokes-Coriolis force play a considerable role in the upper ocean mixing in that area.

In this short account I discuss briefly the following items:

- Mixed layer model

Monin-Obukhov similarity does not work for the upper ocean mixing because surface wave dissipation and Langmuir turbulence produce large deviations from the usual balance between production, buoyancy and dissipation.

- Wave breaking, Langmuir circulation and mixed layer

The energy flux  $\Phi_{oc}$  from atmosphere to ocean is controlled by wave breaking. The result is an energy flux of the form  $\Phi_{oc} = m\rho_a u_*^3$  where  $\rho_a$  is the air density and  $u_*$  is the friction velocity in air. In general the parameter  $m$  depends on the sea state and is not a constant. In order to illustrate this, a plot of the monthly mean of the parameter  $m$  defined as the ratio of the monthly mean of energy flux normalized with the monthly mean of  $\rho_a u_*^3$  is shown in Fig. 1. Both parameters are obtained from the ECMWF ERA-interim results (Dee et al. (2011)). It is clear from the Figure that on average in the Tropics the  $m$ -parameter is small compared to the extra-Tropics. In particular, at the location in the Arabian Sea the atmosphere-ocean energy flux can become anomalously low, suggesting that the mixing might be low as well and therefore there is potential for a large diurnal cycle.

Wave breaking penetrates into the ocean at a scale of the order of the significant wave height  $H_S$ . Also, the shear in the Stokes drift gives an additional production of turbulent kinetic energy which penetrates into the ocean at a scale of the order the typical wavelength of the surface waves.

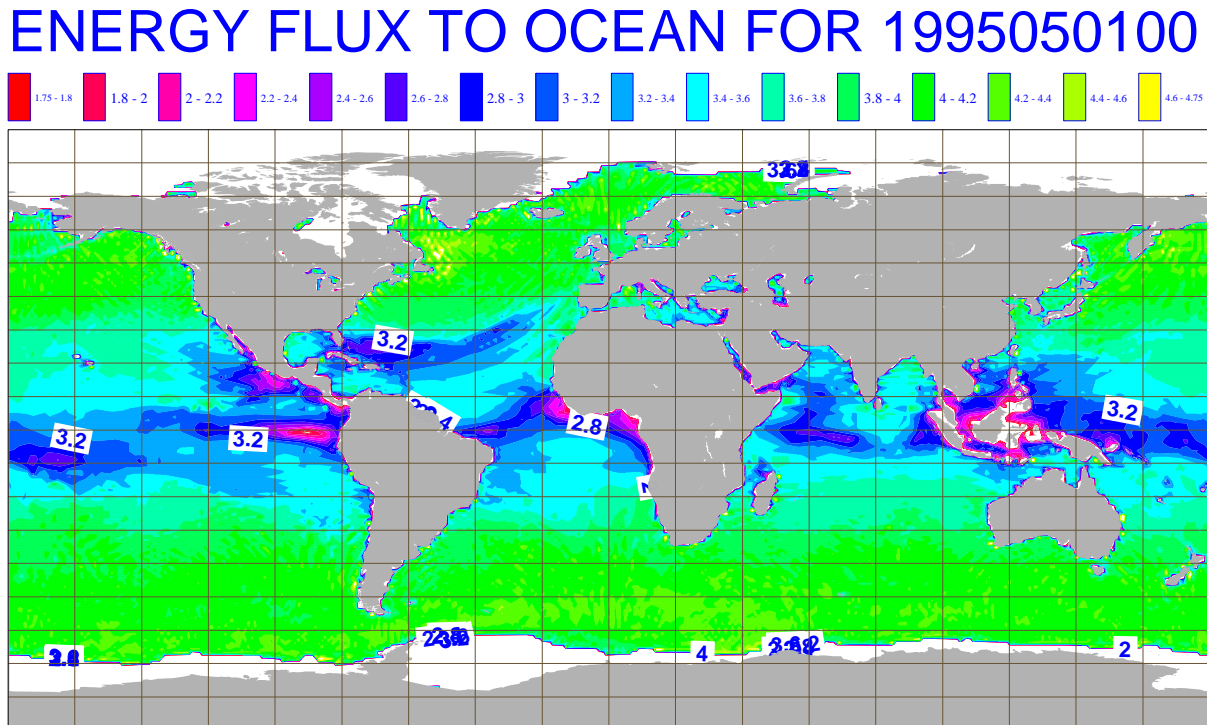


Figure 1: Monthly mean of energy flux into the ocean, normalized with the monthly mean of  $\rho_a u_*^3$ . Period is May 1995.

- Buoyancy effects

Buoyancy effects are essential for modelling diurnal cycle. I have used the approach of [Noh and Kim \(1999\)](#) but adapted the relevant coefficients based on knowledge of stratification effects from the atmospheric community.

- Diurnal cycle in SST

Diurnal cycle in SST follows from the balance between absorption of Solar radiation in the ocean column on the one hand and transport of heat by turbulence on the other hand. Since Solar absorption is fairly well-known, the study of the diurnal cycle is a good test of our ideas of mixing in the upper ocean. Also, using the ASE observations I will show that sea state effects are relevant for upper ocean mixing.

A more detailed account of this work will be published shortly ([Janssen \(2012\)](#)), which discusses topics such as how effects of buoyancy and mixing by dissipating waves and Langmuir turbulence have been obtained. It also describes some of the mathematical derivations of the results. In this account only a brief summary of the main conclusions will be given.

## 2 Mixed Layer Model

Here we briefly describe a multi-layer model of turbulent mixing in the upper ocean that includes effects of surface wave damping, wave-induced turbulence and stratification in addition to the usual shear production and dissipation. A central role in this model is played by the Turbulent Kinetic Energy (TKE) equation, which basically combines all the above physical processes to obtain the TKE as function of depth and time. TKE is then used to determine the eddy viscosities in the momentum and heat equations. Additional effects such as Stokes-Coriolis forcing will be introduced as well.

### 2.1 TKE equation

If effects of advection are ignored, the TKE equation describes the rate of change of turbulent kinetic energy  $e$  due to processes such as shear production (including the shear in the Stokes drift), damping by buoyancy, vertical transport of pressure and TKE, and turbulent dissipation  $\varepsilon$ . It reads

$$\frac{\partial e}{\partial t} = \nu_m S^2 + \nu_m \mathbf{S} \cdot \frac{\partial \mathbf{U}_S}{\partial z} - \nu_h N^2 - \frac{1}{\rho_w} \frac{\partial}{\partial z} (\overline{\delta p \delta w}) - \frac{\partial}{\partial z} (\overline{e \delta w}) - \varepsilon,$$

where  $e = q^2/2$ , with  $q$  the turbulent velocity,  $\mathbf{S} = \partial \mathbf{U} / \partial z$  and  $N^2 = -g \rho_0^{-1} \partial \rho / \partial z$ , with  $N$  the Brunt-Väisälä frequency,  $\rho_w$  is the water density,  $\delta p$  and  $\delta w$  are the pressure and vertical velocity fluctuations and the over-bar denotes an average taken over a time scale that removes linear turbulent fluctuations.

Following [Grant and Belcher \(2009\)](#) and [Huang and Qiao \(2010\)](#) wave-induced turbulence is modelled by introducing work against the shear in the Stokes drift. Here  $U_S$  is the magnitude of the Stokes drift for a general wave spectrum  $F(\omega)$ ,

$$\mathbf{U}_S = 2 \int_0^\infty d\omega \omega \mathbf{k} F(\omega) e^{-2k|z|}, \quad k = \omega^2/g.$$

In stead of this I will use the approximate expression

$$\mathbf{U}_S = \mathbf{U}_S(0) e^{-2k_S|z|},$$

where  $U_S(0)$  is the value of the Stokes drift at the surface and  $k_S$  is an appropriately chosen wavenumber scale.

The dissipation term is taken to be proportional to the cube of the turbulent velocity divided by the mixing length  $l(z)$

$$\varepsilon = \frac{q^3}{Bl},$$

Here,  $B$  is dimensionless constant. The mixing length  $l$  is chosen as the usual one for neutrally stable flow, i.e.  $l(z) = \kappa|z|$ , with  $\kappa = 0.4$  the von Kármán constant.

It is customary (see e.g. Mellor and Yamada (1982)) to model the combined effects of the pressure term and the vertical transport of TKE by means of a diffusion term. However, the pressure term can also be determined by explicitly modelling the energy transport caused by wave breaking. The correlation between pressure fluctuation and vertical velocity fluctuation at the surface is related to the total dissipation of the waves (which follows from the integration of the dissipation sourcefunction  $S_{diss}$  over wavenumber space). Hence

$$I_w(0) = \frac{1}{\rho_w} \overline{\delta p \delta w}(z=0) = g \int_0^\infty S_{diss}(\mathbf{k}) d\mathbf{k} = -m \frac{\rho_a}{\rho_w} u_*^3 = -m \frac{\rho_w^{1/2}}{\rho_a^{1/2}} w_*^3 = -\alpha w_*^3$$

where  $w_*$  is the friction velocity in water. and the main problem is how to model the depth dependence of  $\overline{\delta p \delta w}$ . Assume depth scale is controlled by significant wave height  $H_S$  and that  $I_w(z)$  decays exponentially:

$$I_w(z) = + \frac{1}{\rho_w} \overline{\delta p \delta w} = -\alpha w_*^3 \times \hat{I}_w(z), \quad \hat{I}_w(z) = e^{-|z|/z_0},$$

where the depth scale  $z_0 \sim H_S$  will play the role of a roughness length. Thus, the TKE equation becomes

$$\frac{\partial e}{\partial t} = \frac{\partial}{\partial z} \left( lq S_q \frac{\partial e}{\partial z} \right) - \frac{\partial I_w(z)}{\partial z} + v_m S^2 + v_m \mathbf{S} \cdot \frac{\partial \mathbf{U}_S}{\partial z} - v_h N^2 - \frac{q^3}{Bl(z)}.$$

At the surface there is no direct conversion of mechanical energy to turbulent energy and therefore the turbulent energy flux is assumed to vanish. Hence the boundary conditions become

$$lq S_q \frac{\partial e}{\partial z} = 0 \quad \text{for } z = 0, \quad \text{and} \quad \frac{\partial e}{\partial z} = 0 \quad \text{for } z = -D.$$

where  $D$  denotes the depth of the mixed layer model.

## 2.2 Momentum equation

To simplify the problem, the wind/wave driven water velocity is assumed to be uniform without any pressure gradients in the horizontal directions. Taking into account effects from the wave-induced stresses the momentum equations then reduce to

$$\frac{\partial \mathbf{u}}{\partial t} = \frac{\partial}{\partial z} \left( v_m \frac{\partial \mathbf{u}}{\partial z} \right) + (\mathbf{u} + \mathbf{u}_{Stokes}) \times \mathbf{f}.$$

Here,  $v_m$  is the eddy viscosity for momentum which depends on the turbulent velocity  $q = (2e)^{1/2}$ , i.e.,

$$v_{m,h} = l(z)q(z)S_{M,H}$$

where  $l(z)$  is the turbulent mixing length, and  $S_M$  and  $S_H$  are dimensionless parameters which may still depend on stratification.

Note that the Stokes-Coriolis force is obtained from the radiation stress on a rotating sphere.

### 2.3 Heat Equation

The heat equation describes the evolution of the temperature  $T$  due to radiative forcing and turbulent diffusion. Using the depth variable  $z$ , the temperature evolves according to

$$\frac{\partial T}{\partial t} = -\frac{1}{\rho_w c_w} \frac{\partial R}{\partial z} + \frac{\partial}{\partial z} v_h \frac{\partial T}{\partial z},$$

where  $v_h$  is the eddy viscosity for heat, while the solar radiation profile  $R(z)$  is parametrized following the work of Soloviev (1982), i.e.

$$R(z) = a_1 \exp(z/z_1) + a_2 \exp(z/z_2) + a_3 \exp(z/z_3), \quad z < 0,$$

with

$$(a_1, a_2, a_3) = (0.28, 0.27, 0.45) \text{ and } (z_1, z_2, z_3) = (0.013986, 0.357143, 14.28571).$$

### 2.4 Buoyancy

Extreme events in diurnal cycle typically arise for low winds. Buoyancy effects are then important, because they reduce mixing giving rise to heating up of the top layer of the ocean.

Effects of stratification are modelled using the approach of Noh and Kim (1999). Under very stable conditions one would expect that turbulence is characterized by the Brunt-Väisälä frequency'  $N$  ( $N^2 = -g\rho_0^{-1} \partial\rho/\partial z$ ). This suggests that the mixing length is limited by an additional length scale  $l_b = q/N$ . The eddy viscosity can then be estimated by

$$\nu \sim ql_b \sim ql Ri_t^{-1/2}$$

where

$$Ri_t = (Nl/q)^2$$

is the Richardson number for turbulent eddies and  $l$  the mixing length.

## 3 Steady state properties for neutrally stable flow

The properties of the steady state version of the TKE equation were studied extensively. In case turbulent transport of TKE can be ignored it is possible to obtain an approximate solution, which is completely determined by the local properties of the flow, hence the name the 'local' approximation. Without presenting any of the details (see for this Janssen (2012), for neutral stratification the following '**1/3<sup>rd</sup>-rule**' is found. Introducing the dimensionless turbulent velocity  $Q = (S_M/B)^{1/4} \times q/w_*$  the approximate solution of the TKE equation becomes

$$w(z) = Q^3 \approx 1 + \alpha \kappa |z| \frac{d\hat{I}_w}{dz} + La^{-2} \kappa |z| \frac{d\hat{U}_S}{dz},$$

where  $La = (w_*/U_S(0))^{1/2}$  is the turbulent Langmuir number and  $\hat{U}_S$  is the Stokes drift profile. So in terms of  $Q^3$  there is a **superposition principle** of physical processes, i.e. contributions due to wave dissipation and Langmuir turbulence may be added to the shear production term.

Fig. 2 shows the contributions of wave dissipation and Langmuir turbulence to the turbulent velocity. It is clear from this picture that mixing by wave dissipation mainly occurs very close to the surface, at

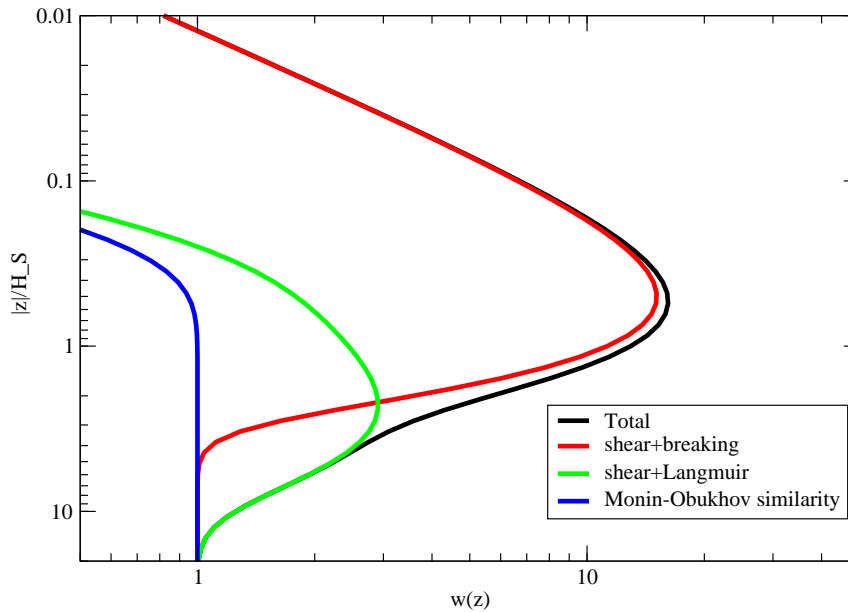


Figure 2: Profile of  $w = Q^3$  according to the local approximation in the ocean column near the surface. The contributions by wave dissipation (red line) and Langmuir turbulence (green line) are shown as well. Finally, the  $w$ -profile according to Monin-Obukhov similarity, which is basically the balance between shear production and dissipation, is shown as the blue line.

a depth of the order of the wave height  $H_S$ , while Langmuir turbulence has its maximum impact at the larger depth of  $1/k_s$ . These scales are widely different because ocean waves are weakly nonlinear which means that their characteristic steepness  $k_s H_S$  is small, of the order of 10%. As a consequence the ratio of the penetration depths by wave dissipation and Langmuir turbulence, given, by  $k_s H_S$ , is small as well.

Furthermore the solution in the local approximation helps to understand the sensitivity of upper ocean transport to variability in the sea state. When determining the energy flux from the dissipating waves it is found that there is high variability in the dimensionless flux  $\alpha$  in particular near the passage of a front. Hence, as the turbulent velocity depends, according to the '1/3'-rule, on  $\alpha^{1/3}$ , its variability and the variability in the turbulent transport is much reduced. The '1/3'-rule also explains that when only Langmuir turbulence is taken into account the turbulent velocity scales with  $La^{-2/3}$  in agreement with the scaling arguments of [Grant and Belcher \(2009\)](#).

## 4 Simulation of the diurnal cycle

The mixed layer model was solved using the boundary condition that at depth  $D = 20$  m current velocity  $u(z)$ ,  $v(z)$  and temperature  $T(z)$  are given. The equations for momentum, heat and turbulent kinetic energy are discretized in such a way that the fluxes are conserved, while the quantities are advanced in time using an implicit scheme with a time step of 5 minutes. The vertical discretization is obtained using a logarithmic transformation. Because such a transformation is so efficient in capturing the relevant details only 25 layers are required.

The model was driven by observations of fluxes and solar insolation for a period of one year at a location at  $15^\circ 30'$  N,  $61^\circ 30'$  E in the Arabian Sea (The Arabian Sea Experiment, cf. [Baumgartner et al. \(1997\)](#); [Weller et al. \(2002\)](#)). The ocean wave parameters were obtained from wave spectra produced by the

ERA-Interim reanalysis (Dee et al. (2011)).

Modelled Diurnal SST Amplitude (DSA) is compared with observations in Fig. 3. A good agreement is obtained, while errors are bigger when wave effects are ignored.

Finally, the ASE has a large volume of high quality observations. I took the opportunity to validate the modelled temperature profiles against the hourly observations. Results of the depth-dependent bias and standard deviation of error are shown in Fig. 4. This allowed me to check the impact of the wave breaking parametrization and the Stokes-Coriolis force.

- Wave breaking parametrization results in a considerable reduction of errors near the surface (depth  $\simeq H_S$ ).
- Stokes-Coriolis force gives a positive impact in the deeper layers of the ocean ( $\simeq 1/k$ )
- No impact of Langmuir turbulence was found. Although the average turbulent Langmuir number  $La \simeq 0.4$  suggesting that mixing by Langmuir turbulence should occur no sensitive dependence of the results for SST on turbulent Langmuir mixing at this site was found. Langmuir turbulence acts in the deeper layers of the ocean where in particular during the day time buoyancy effects may have a stabilizing influence on the turbulent production. In this respect, it should be noted that mixing by wave dissipation is less affected by stratification effects as this process occurs much closer to the surface.

## 5 Conclusions

My conclusions are the following:

- Wave effects, such as wave breaking and Stokes-Coriolis forcing, are important even under low-wind conditions.
- Wave breaking and buoyancy play a key role in understanding the diurnal cycle in SST and in the current.

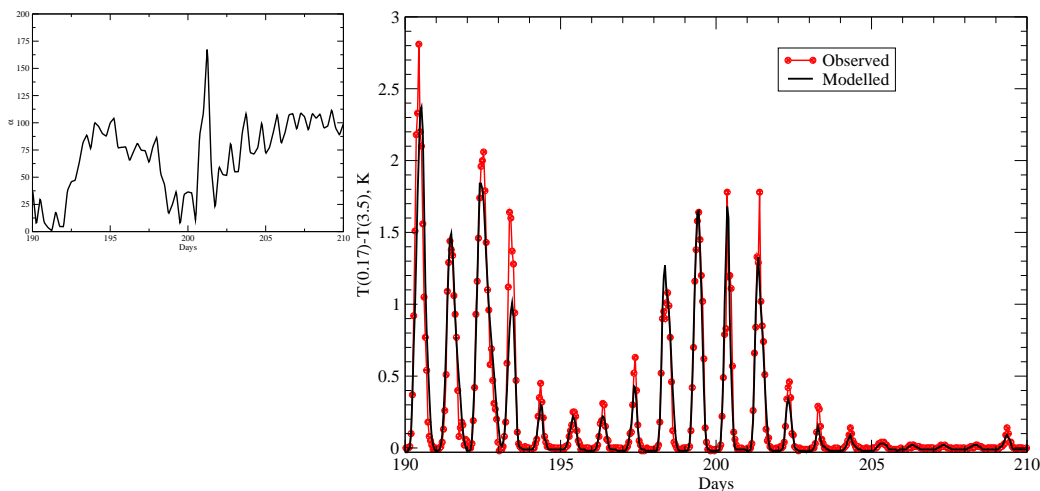


Figure 3: Left Panel: Timeseries of normalized energy flux  $\alpha$ . Right Panel: Observed and simulated ocean temperature  $\Delta T = T(0.17) - T(3.5)$  at  $15^{\circ}30' N$ ,  $61^{\circ}30' E$  in the Arabian Sea for 20 days from the 23rd of April.

- Simulations of SST are possible with high accuracy as the random error is just above 0.15 K.

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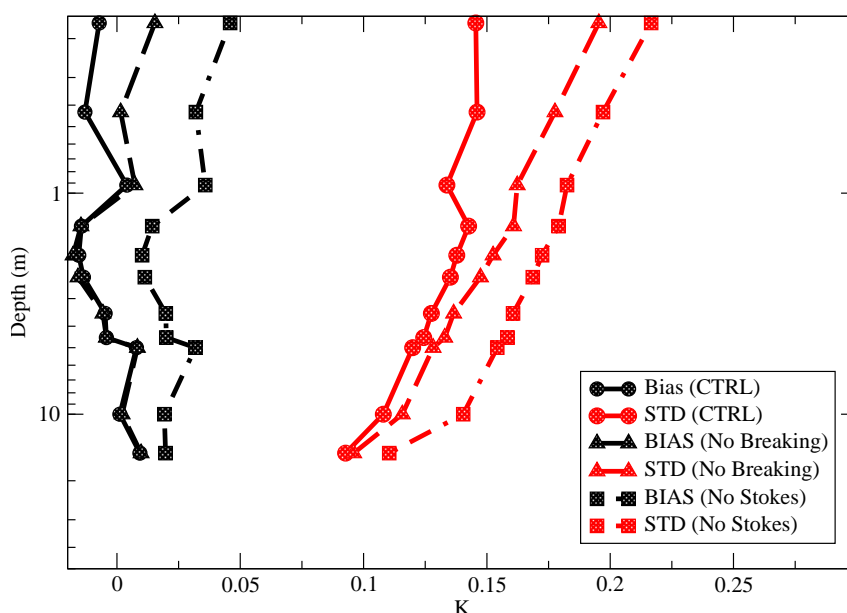


Figure 4: Comparison of modelled temperature data against hourly observations as function of depth at 15°30' N, 61°30' E in the Arabian Sea for one year from the 16th of October 1994.



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