Moist thermodynamics and moist turbulence for modelling at the non-hydrostatic scales

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Abstract

The improvement of Non-hydrostatic modelling is addressed with the perspective of using a multi-phasic and barycentric framework for the moist atmosphere, with the hypothesis that an enhanced consistency of the moist thermodynamic equations may lead to larger impacts than the pure Non-hydrostatic pressure force problem. The possibility to use improved conservative laws is also examined, starting with a new definition for the entropy of the moist atmosphere.

1. Introduction

This note has to do with the diabatic part of high resolution modelling (and with its 'interfacing'). There is no basic distinction between hydrostatic and non-hydrostatic modelling. Indeed the issue about the need (or not) of a treatment differing in the non-hydrostatic case from the one of the hydrostatic system is left to the paper by S. Malardel in the same Proceedings volume. We are however mostly interested in the scales where using the non-hydrostatic equations is quasi-compulsory. Hence, '*Non-Hydrostatic*' (NH) is here just a short-hand for 'applicable at scales where non-hydrostatism matters for reversible motions'.

It is our experience that at such high (horizontal) resolutions (let us say below δx~3km):

- The devil is really in the detail … (two examples will be shown later rather extensively);
- It becomes crucial for the reliability of the results to use quite firm modelling guidelines;
- There is a need for new unifying concepts and probably for revisiting some long-lasting paradigms.

In this perspective, the key-words of this note will be: 'conservation laws', 'consistency', 'entropy' and 'multi-phasic systems'. Anticipating a bit one conclusion of this study, we may already say that the most noticeable feature, for diabatic processes' handling, when reaching the 'NH scales', is the strong increase of sensitivity to a lot of detailed choices, rather than some reaction to the presence of new modes in the adiabatic part of the model. This is contrary to first intuition, but quite obvious in (pre-) operational practice. This recognition however does not give any good guideline for attacking the issue in a particular way.

Here we are therefore electing to concentrate on two questions (leaving the rest for a mere enumeration in the outlook part):

- How to represent pressure gradients in multi-phasic systems (both horizontally and vertically)?

- Which thermodynamic quantity's conservation law is most appropriate as guideline, when 'subgrid' reduces more and more to 'turbulent + cloudy vs. clear sky'?

And our answers will be (under the ever present guideline of '*consistency*'):

- By using a barycentric framework for developing the generic conservation equations of the 'physics-dynamics' interface;

- Entropy, for several reasons.

The reasoning will be presented as follows. In the next Section we shall provide a simplified but fully consistent framework for the thermodynamic set of equations that ought to govern the interaction between the diabatic and adiabatic parts of any model for which a numerical transcription of the conservation laws in nature is judged important. The ensuing Sections will present the above-mentioned two examples of details' importance for NWP predictions at NH scales. We shall then go to a wider perspective concerning moist entropy, showing (in two Sections) a link between basic considerations and some observational fact, before extrapolating toward a (possible) renewed view of parameterisation of moist turbulence. A mix of Conclusions and Outlook will wrap up this note.

2. A framework for consistent analytical computations of thermodynamic conservation laws

2.1. Simplifying hypotheses

We first need a set of simplifying assumptions (the full problem, with all details as observed in nature is indeed quasi-intractable for NWP purposes). These hypotheses are chosen to be the minimum ones necessary for having conservation laws very close to the (too complex) natural ones, while still allowing an in-depth analytical treatment of all main transport and mixing processes.

For the diabatic part of the model computations (of course with some influence on the equations for the adiabatic 'backbone') we assume that:

- The atmosphere is in permanent thermodynamic equilibrium;

- Condensed phases have a zero volume;

- All gases obey Boyle-Mariotte's and Dalton's laws;

- The specific heat values are all temperature-independent;

- There is local homogeneity of the temperature between all species of an atmospheric parcel.

The first four hypotheses are rather 'classical', even if not always stated in model descriptions. Only the last one is 'oriented' (towards the idea of a 'flux divergence' representation of conservation laws, see below).

Note that we did these 'diabatic' choices independently of issues that more specifically touch the dynamics. Here the rule is rather to have optional choices which dictate the shape of the equations in case the atmosphere would only been made of dry air. For instance the choices are between:

- Hydrostatic Primitive Equations vs. Non-Hydrostatism;
- Conservation of total mass or not, when accounting for the balance between precipitation minus evaporation at the earth's surface (i.e. do we need or not a fictitious counter flux of dry air?);
- Which 'atmospheric parcels' are considered (i.e. do we consider that falling hydrometeors are part of the medium for which we are writing the adiabatic framework or not)?

At that stage one may already point out a practical difficulty for people developing and/or maintaining models: ideally the concretisation of the five diabatic simplifying hypotheses should happen transparently with respect to the 'adiabatic' basic choices. This is possible, but it requires some 'political willingness' of having a 'clean' system for the R&D actions using the model at stake.

2.2. A graphical example of mathematical transcription (a bit simplified for the sake of the presentation)

$$
\begin{cases}
p = \rho . R . T & (Perfect\ gas\ law) \\
R = R_d \, (1 - q_v - q_l - q_i) + R_v \, .q_v & \\
C_p = C_{pd} (1 - q_v - q_l - q_i) + C_{pv} .q_v + C_l .q_l + C_i .q_i & \\
L_{v/s}(T) = L_{v/s}^{T=0} + (C_{pv} - C_{l/i}).T & \\
\dfrac{\partial \ln(e_s(T))}{\partial T} = \dfrac{L_{v/s}(T)}{R_v .T^2} & (Clausius - Clapeyron)
\end{cases}
$$

The notations are as follows:

- ***T*** is the temperature;
- q_v, q_l and q_i are the specific amounts of vapour, liquid and ice phase of water (for this 'local' consideration, q_l encompasses the rain drops and q_i the snow flakes);
- ***p*** is the pressure, ***e*** the water vapour partial pressure (with e_s its value at saturation) and ρ the total density;
- R_d and R_v are the gas constants for respectively dry air and water vapour;
- C_{pd}, C_{pv}, C_l and C_i are the specific heats at constant pressure for respectively dry air, water vapour, liquid water and ice water;
- $L_v(T=0)$ and $L_s(T=0)$ are the latent heat values at ***0*** degree Kelvin for respectively vaporisation and sublimation. Note that these values are not true latent heat values at $T=0$. They represent a

shortcut notation for the extrapolation toward $T=0$ of $L_v(T)$ and $L_s(T)$ if the actual atmospheric conditions would prevail at low temperature (i.e. for the same constant values of C_{pv}, C_l and C_i).

R, C_p, L_v and L_s can be deduced from the above set of equations, for any values of the prognostic quantities T, q_v, q_l and q_i.

The coloured boxes and the arrows indicate the links between the various equations. The blank part at the end of the derivation of 'R' corresponds to the hypothesis of 'zero volume for condensates'. It should be mentioned that one of the most important constraints for benefitting from the consistency of the above system is to couple the C_p (and R, hence also C_v) dependency on $q_{v/l/i}$ with the $L_{v/s}$ dependency on T.

One interesting consequence of this set of equations (put together with some more advanced thermodynamic definitions) is that it allows writing a compact non-approximated form for the entropy of moist air (including condensates), conservative under the conditions of no radiative source/sink and no turbulent sink of energy and expressed only with the physical constants present in the above set of equations (for details see Marquet (1993)):

$$s_m = (C_{pd} + r_t.C_{pv}).\ln(T) - R_d.\ln(p-e) - r_t.R_v.\ln(e) - \frac{L_v(T).r_l + L_s(T).r_i}{T}$$
$$\textit{with} \quad r_{(\_)} = q_{(\_)}/(1-q_v-q_l-q_i) \qquad \textit{and} \quad r_t = r_v + r_l + r_i$$

We shall come back later to another interesting property of this moist entropy formulation, but in between we shall concentrate on issues arising from the above definitions of respectively R and C_p.

3. The so-called 'LSPRT issue'

3.1. A big influence for a supposedly small simplification

The topic addressed here is specific to spectral modelling, but the 'message' we shall deliver is, in our opinion, of far wider interest. Let us first explain the genesis of the technical problem.

The computation of the horizontal gradient of geopotential as a contribution to the pressure gradient term requires using only horizontal derivatives of the prognostic variables of any model. In case of a spectral model, so-called 'grid-point variables' (hydrometeors typically) which do not have a spectral representation cannot enter this computation if temperature T is the thermodynamic prognostic spectral variable.

In such a case (and provided q_v is treated spectrally) the computation is approximated by using $R=R_d+q_v(R_v-R_d)$. But, if we use the correct equation for the pressure thickness of a layer, in the vertical, we get $d(\Phi)=-[R_d(1-q_v-q_l-q_i-q_r-q_s)+R_v.q_v].T.d(\ln(p))$. This situation corresponds in the IFS/ARPEGE/ALADIN framework to the logical option LSPRT being set to 'false'.

On the contrary, activating the LSPRT option as 'true' makes RT (an equivalent to the virtual temperature) the thermodynamic prognostic variable of the model in spectral space. The choice of R is then not any more restricted by spectral considerations for the horizontal part of the computations and the latter can be made fully compatible with the corresponding vertical part, concerning the tri-dimensional gradient of geopotential.

The order of magnitude of the discrepancy between both solutions appears a-priori small, since it corresponds to the sum of the specific mass of all condensates. In integrations at larger scales than the NH one (down to δx~5km), no systematic impact could be detected (not shown) and the choice between the two solutions was left to other practical considerations than the one of consistency between the horizontal and vertical expressions for the gradient of geopotential.

However, after some AROME results drew attention to the impact of changing the value of the LSPRT switch (Malardel and Bouteloup, personal communication) comparing the two solutions in a case of relatively heavy precipitations over Central Europe at 2km mesh-size delivers a completely new message. To make the comparison as meaningful as possible the LSPRT 'true' option is activated with $\boldsymbol{q_v}$ still treated as spectral variable (fortunately this combination is allowed). In areas of weak precipitations not much happens, but heavy local maxima are systematically reinforced when using the inconsistent solution, by a factor that may reach two (see Figure 1). One may imagine that at hectometric scales these already impressive results may be superseded by even more spectacular ones, the diverging solution (from scale to scale) being most likely the one of LSPRT set to 'false'.

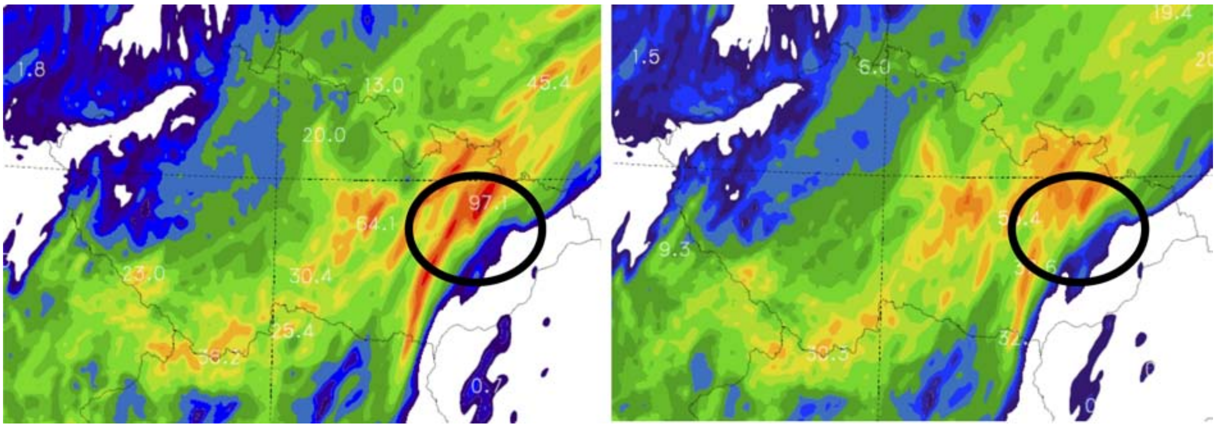

*Figure 1: 6h cumulated precipitation amounts (12h – 18h) focussed over the area of the Czech Republic for a forecast at 2.3 km mesh-size with the ALARO-0 version of ALADIN starting on 00 UTC 18/05/08. **Left panel**: with LSPRT=.FALSE. (inconsistent treatment of the 3D gradient of geopotential). **Right panel**: with LSPRT=.TRUE. (consistent treatment of the 3D gradient of geopotential). In the marked area of maximum discrepancy the maximum is 97.1 mm on the left and 55.4 mm on the right. As a matter of comparison the highest local estimate of a radar+raingage composite data set gives 46 mm in the relevant area. Courtesy of R. Brožková.*

As already hinted at, the lesson of this surprising result extends well beyond the specific spectral framework that allowed discovering and quantifying it. In a nutshell, at NH scales, even some small inconsistencies in the formulation of the interplay between physics and dynamics with respect to prognostic equations for condensed species may have a dramatic impact on the whole hydrological cycle. This happens in case of strong events, prone to sensitivity to potential positive feed back loops. One may of course discard these results as 'unrepresentative' of the general model behaviour. But (i) the prediction of intense events is a target per se and (b) when one might easily ensure consistency of a numerical formulation, why sticking to another solution for the mere comfort of not having to revalidate a model configuration?

3.2. What is behind the choice of a fully multi-phasic 'R' value?

We have just seen the key role, at the 'NH scales', of the choice of '$\boldsymbol{R}$'. In the shown example, the falling species $q_{r/s}$ (for 'rain' and 'snow') were also accounted for, in a prognostic manner. This corresponds to the choice of the so-called 'barycentric' definition of the 'parcel' (precipitation becomes another sub-grid, just better organised, transport). The (non barycentric and rather 'classical') alternative is to exclude $q_{r/s}$ from what the adiabatic part of the model 'sees' and to treat these species separately. This is quite easy when dealing with their 'steady' regime of fall, but what about their acceleration phase and/or their sudden disappearance through evaporation-sublimation?

In nature, what prevents condensed species from reaching higher and higher fall-speeds is a local pressure gradient between the top and bottom of drops/crystals, a gradient also felt in the whole atmospheric column by continuity of the pressure field.

Hence, in the case of the hydrostatic assumption (and of a prognostic treatment of $q_{r/s}$) it is fully correct to assume that $dp=-\rho.d\Phi$ must be computed with ρ accounting for the presence of falling species. In the case of using the barycentric equations, this choice 'filters out' the issue about local volume changes when condensed water species do appear and/or disappear.

In the NH case, one can show (Catry et al., 2007) that the above 'filtering condition' becomes $p = \rho_{gas}R_{gas}T = \rho RT$ (with R and ρ taking all species into account). When going back to the non-barycentric system, the filtering disappears for $q_{r/s}$ and one should in principle account for their acceleration phase as well as for their return to vapour! This being a paramount task, the choice is rather between neglecting small terms or doing the effort to write the equations in a barycentric framework and to make this interact correctly with the diabatic simplifying hypotheses. Given the above demonstration of the danger of having 'small' accepted approximations in this link between the continuity equation and the gradients of the geopotential, our strong recommendation is to go for the barycentric solution, latest when approaching the NH scales.

4. Green-Ostrogradsky form of the thermodynamic equation and another related surprising result

The following will have to do with the intra-time-step variations of C_p, C_v and hence R, following the phase changes of a barycentric multi-phasic system (here $q_{v/l/i/r/s}$ are considered). The type of unwanted simplification we wish to trace is not the very crude identification of C_p with C_{pd}, but rather the fact to have a correctly defined C_p assumed not to evolve during the 'physics time step of the model' (an approximation often made 'for convenience').

Starting from the above conservation law for moist entropy, using $C_p=C_v+R$ and the first principle of thermodynamics one gets (Catry et al., 2007) a Green-Ostrogradsky form for the evolution of a particular form of the model enthalpy:

$$\frac{d}{dt}(C_pT) = g\frac{\partial}{\partial p}[\, L_v(T=0)(P_l' - P_l''') - (C_l - C_{pd})P_lT$$
$$+ L_s(T=0)(P_i' - P_i''') - (C_i - C_{pd})P_iT$$
$$+ \delta_m\left(\frac{C_{pd}q_d + C_{pv}q_v + C_lq_l + C_iq_i}{1 - q_r - q_s} - C_{pd}\right)(P_l + P_i)T$$
$$-(J_s + R_{ad})\,]$$

The notations are as follows:

- $\boldsymbol{P_l}$, $\boldsymbol{P_i}$ are, respectively for the liquid and ice phase, the precipitation fluxes;
- $\boldsymbol{P'_l}$, $\boldsymbol{P'_i}$ are, respectively for the liquid and ice phase, the pseudo-fluxes which divergences correspond to the local (in cloud) condensation minus evaporation processes;
- $\boldsymbol{P'''_l}$, $\boldsymbol{P'''_i}$ are, respectively for the liquid and ice phase, the pseudo-fluxes which divergences correspond to the evaporation processes of falling species;
- In the two above cases, any phase change happening directly between condensed phases (alike melting in the falling phase) can be thermodynamically identified as a combination of the processes described by $\boldsymbol{P'_l}$, $\boldsymbol{P'''_l}$, $\boldsymbol{P'_i}$ and $\boldsymbol{P'''_i}$;
- $\boldsymbol{\delta_m}$ is a tag for conservation (***0***) or not (***1***) of the total mass under the influence of the surface balance between precipitation and evaporation; in case it is zero, the barycentric compensation for the moisture transport is ensured by a fictitious flux of dry air; in case it is one, this compensation is ensured by a real flux of the non-precipitating part of the atmospheric parcel;
- $\boldsymbol{J_s}$ is the diffusive flux of dry static energy ('sensible heat') and $\boldsymbol{R_{ad}}$ the net (solar + thermal) radiative flux;
- All fluxes and pseudo-fluxes are counted positively downwards.

The conservation law for enthalpy must in principle be written as ***dh/dt=0***, with ***h*** different from $\boldsymbol{C_pT}$ for the moist atmosphere. However, the conservation law for ***h*** can be written in terms of $\boldsymbol{d(C_p.T)/dt}$ (or $\boldsymbol{C_p.dT/dt}$ with another right hand side), if the relevant additional terms are taken into account (like in Marquet, 1993 and in Catry et al., 2007), leading to the above formula expressed with a full flux divergence on the right hand side.

The use of the shortcuts $\boldsymbol{L_v(T=0)}$ and $\boldsymbol{L_s(T=0)}$ in the above equation is indeed coupled with the accounting of the differential heat transport (owing to $\boldsymbol{C_l}$ and $\boldsymbol{C_i}$ different from the $\boldsymbol{C_p}$ of the compensating motion) by falling species. If either the differential heat transport is neglected (like often advocated for the sake of simplicity) or $\boldsymbol{L_s}$ and $\boldsymbol{L_v}$ are supposed to be temperature independent, the property of integral conservation of enthalpy is therefore lost.

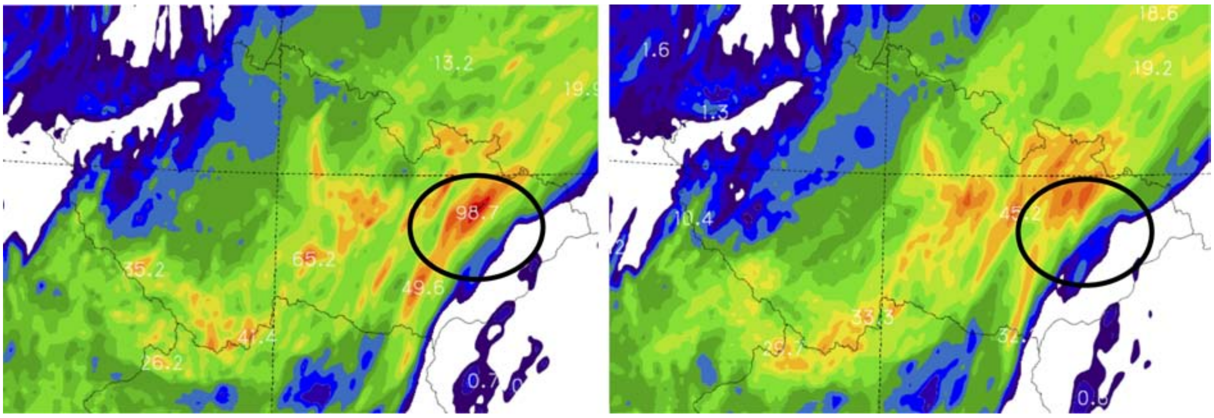

*Figure 2: As Figure 1 for the situation and the basic set-up (both runs are made here with a more recent version of the model, this explaining the change in the 'reference' result). **Left panel**: without enthalpy conservation (see text). **Right panel**: with enthalpy conservation. In the marked area of maximum discrepancy the maximum is 98.7 mm on the left and 45.2 mm on the right. As a matter of comparison the highest local estimate of a radar+raingage composite data set gives 46 mm in the relevant area. Courtesy of R. Brožková*

It is sometimes customary to say that neglecting the time variation of C_p (or C_v, or R) during the 'physical time-step' (under the influence of phase changes) has little impact. Like in the 'LSPRT' case, we shall now see that this is not true at all at the 'NH scales'. The trick for making the cleanest possible test, given the compact shape of the previous flux-conservative form of the enthalpy equation, is just to replace on the left-hand side ' $d(C_p.T)$ ' by ' $C_p.dT$ '! The result is shown on Figure 2 and resembles much the one of Figure 1! Here also it should be noted that such a high sensitivity is only visible at the NH scales, even if the transition is more progressive than in the previous case (already some signs at the 5km mesh-size, not shown).

It should be mentioned that the above test is somewhat extreme. Less serious discrepancies can be expected if the non-accounting of $d(C_p)$ during the time step is linked with some special choices for the latent heat terms and for the way to account (or not) for the heat transport by falling species. But, if such solutions can be used to avoid having the damaging consequences seen on Figure 2 (worse that those of the inconsistency in the 3D geopotential gradient), one is then clearly relying on intentional compensating errors to avoid the consequences of a basic unrealistic choice. In our opinion, doing the effort to ensure the integral conservation of enthalpy via a correct local budget of entropy is highly preferable.

5. The Moist Entropic Potential Temperature

The well-known Betts' moist potential temperature θ_l is usually considered to verify both good 'Lagrangian' and 'intensive' conservation properties. In fact, θ_l has been defined in 1973 by Betts as an approximated synonym of the moist entropy (s_m), from which approximate conservative properties automatically ensue. Given the nice link between 'local' moist entropy conservation and 'integral' enthalpy balanced budgets, any other more interesting synonym of the moist entropy could possess new attractive conservation properties.

The aim of a recent new proposal (Marquet, 2011) is to come back to the definition of the moist entropy and to revisit the way a moist potential temperature could be derived from it. A new moist potential temperature is defined. It is denoted by θ_s. The main advantage of θ_s is that all the variations of the total water content q_t (the second of Betts' variables) are taken into account in both formulations for s_m and for θ_s, whereas approximations do exist in the Betts' paper. Additionally, unlike earlier proposals, the value of θ_s is per definition fully independent of the reference values needed in the entropic computations. As will be hinted at in Section 6, comparison with in-situ measurements seems to give a strong credibility to this new way of defining an entropic potential temperature.

A first order approximation for θ_s is denoted by $(\theta_s)_1$. The accuracy of $(\theta_s)_1$ is better than 0.6K and it is surprisingly interesting that it represents a combination of both Betts variables, with:

$$\theta_s \approx \left(\theta_s\right)_1 = \theta_l . \exp(\Lambda . q_t)$$

The non-dimensional parameter $\Lambda = (s_d - s_v) / C_{pd} \approx \mathbf{5.87}$ depends on the absolute entropies of the dry air and of the water vapour. In some sense, while s_m can be computed using physical constants linked with the first principle of thermodynamics (see above), the balance between heat and moisture components within $(\theta_s)_1$ requires a quantity related to the second principle.

6. Verification on FIRE-I data

The marine Stratocumulus (Sc) is the paradigm of the moist turbulence. It is of common use to plot the vertical profiles of the Betts 'conservative' variables to verify their constant feature, with the Betts' variables becoming the internal variables of most of the turbulent schemes.

The FIRE_I data flights have been used to compute the vertical profiles of the moist potential temperatures θ_l and $(\theta_s)_1$, including the standard variations represented by small horizontal bars on the left panel of Figure 3. It is shown that the in-cloud and the clear-air values of θ_l are differing from each other in both the cloud- and the entrainment regions.

The vertical profiles of $(\theta_s)_1$ are more homogeneous, for both the cloudy and the clear air parts, with small vertical gradient of $(\theta_s)_1$ within the whole PBL and with the same value for the clear-air and the in-cloud means of the moist entropic potential temperatures. It is important to notice that the 'top of PBL discontinuity' practically disappears when using the new quantity, whereas it is as large as 10K for θ_l.

It may be difficult to accept that (or to understand why) the famous 'top of PBL discontinuity' observed with θ_l could be removed with the use of θ_s or $(\theta_s)_1$. In fact, it is at least possible to understand how these two features may be observed at the same time.

A graphical 3D representation is suggested in the right panel of the Figure 3. The 3D blue curve is built with the plotting of $(\theta_s)_1$ in terms of the coordinates (θ_l, q_t, z). The isentropic lines are represented as slantwise greyish lines on the bottom conserved variables' plane (θ_l, q_t). If the vertical 3D blue curve of $(\theta_s)_1(z)$ is projected along the direction of the vertical isentropic plane, the resulting projected line is almost constant and with almost no 'top of PBL discontinuity', simply because the blue curve is almost parallel to the vertical moist isentropic plane: the blue curve is seen 'in profile' for determining the moist entropy. On the contrary, when the blue curve is projected onto the left and the rear planes, the well-known vertical profiles of θ_l and q_t appear, with the usual large 'top of PBL discontinuities'.

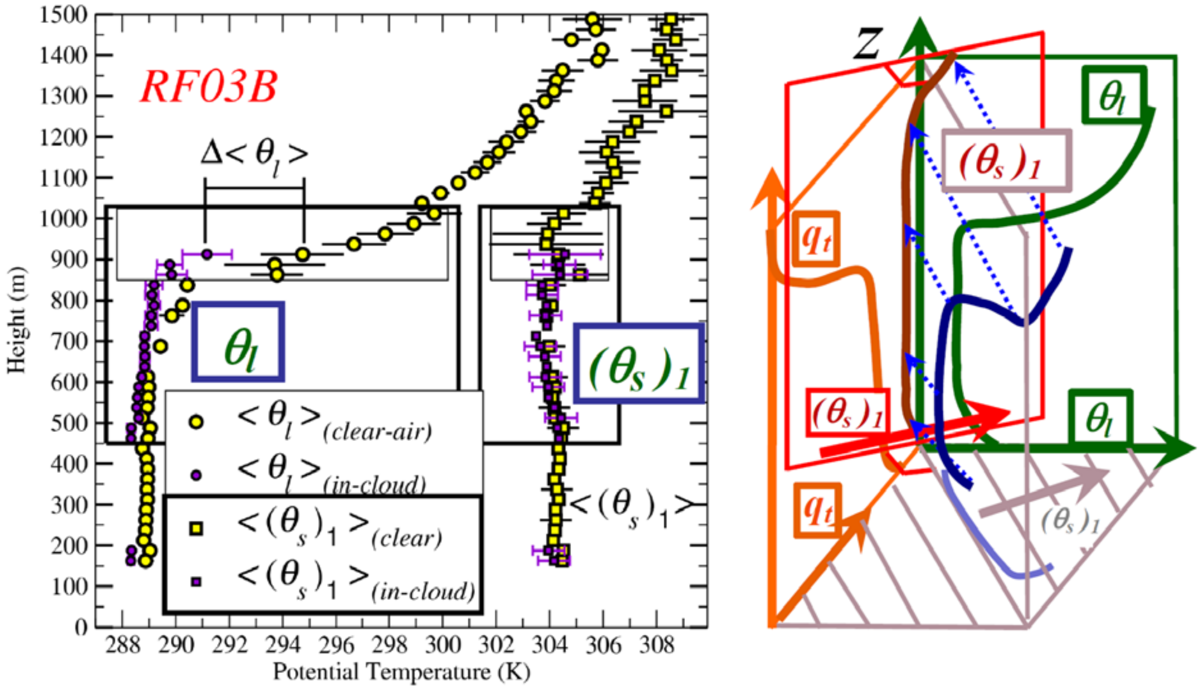

*Figure 3: **Left panel**: The vertical profiles for the clear-air mean values (yellow open symbols) and the in-cloud mean values (purple dark symbols). The Betts potential temperature is plotted on the left (circles) and the new moist entropic potential temperature is plotted on the right (squares). The rectangular windows represent the cloud regions (heavy black lines) and the entrainment regions (light black lines). **Right panel**: A 3D representation of the links between the various involved 'conservative' quantities. The (blue) 'state' curve (in the 3D space of z and of the two moist-conserved variables q_t and θ_l) may be projected in terms of q_t (left / orange) and θ_l (rear / green) but also of their 'combination' $(\theta_s)_1$ (slantwise / red-brown). The 'top of PBL discontinuity', indeed present in the first two cases, disappears in the last one!*

7. Some thoughts for the possible use of the Moist Entropic Potential Temperature in turbulence computations

7.1. Link with the buoyancy term

We now have a moist potential temperature $\boldsymbol{\theta_s}$ conservative for reversible and adiabatic processes, including all those linked to phase changes and showing homogeneous distributions, at least for the Sc case where it seems to be true along both the vertical and the horizontal directions.

What is here at stake is not the fact that the new potential temperature $\boldsymbol{(\theta_s)_1}$ may be as homogeneous in all types of clouds as in Sc. Of course, those Sc are really the ideal case for entropy conservation, but even for the fully dry air case the homogeneity of dry entropy is also used as a relaxation target, even if diabatic sources of heat do exist at the surface! What counts is that the equilibrium position towards which 'moist turbulence' will tend is the one corresponding to this 'well mixed $\boldsymbol{(\theta_s)_1}$ case', even if other diabatic or dynamic terms exist and will tend to create sources or sinks of $\boldsymbol{(\theta_s)_1}$, with therefore the creation of vertical gradients of this quantity also.

More precisely, most of the present turbulent schemes are built with the vertical fluxes of the Betts' variables as input of the thermal production term (even if assembled in the vertical flux of the Lilly's virtual potential temperature $\boldsymbol{\theta_{vl}}$). On one hand, it means that the turbulence schemes are not active for regions where both $\boldsymbol{\theta_l}$ and $\boldsymbol{q_t}$ are constant (cloudiness => 1). On the other hand, they are acting in the

Cumulus or thermal clear-air regions still at the extent of the vertical fluxes of θ_l and q_t (cloudiness => 0). The interest to use $(\theta_s)_1$ in some way, yet to be determined, would be to better represent (and control) the fluxes in terms of moist entropy, which are directly influenced by the diabatic heating rates, in particular for partial cloudiness cases. There is however no obvious way of doing so in the line of the methods employed up to now. This can be understood if one considers that those methods rely on weighting fully dry and fully saturated results with the equivalent of a cloud-cover. But the nice property of $(\theta_s)_1$ to be horizontally homogeneous between clear and cloudy air areas makes it quite a bad tracer of cloudiness (at least for the important case of Sc clouds)!

Said differently, the main difficulty is the need to consider at the same time the constraints of conservation of the moist entropy (made easier with $(\theta_s)_1$) and to take account of the conversion term (or the buoyancy effect). For those latter processes, the use of $(\theta_s)_1$ is still unclear. Some hints may be found in the search of the underlying processes that can explain why the values (and the gradients) of $(\theta_s)_1$ must be the same in the cloudy and clear air areas of marine Sc? Alternatively, considerations separating the energy conversion and entropy production aspects of moist turbulent motions may help solving the problem.

7.2. A new diagnostic of conditional instability

But before attacking such ambitious goals there is an immediate consequence of the results mentioned in Section 6 which can be used for improving existing parameterization schemes of moist turbulence. It is also shown in Marquet (2011) that the quasi neutral PBL behaviour of $(\theta_s)_1$ makes it a likely better diagnostic of true moist conditional instability than the 'classical' variants of the θ_E potential temperature (those apparently leading to a too unstable view, at least in the Sc case). But for such a goal, the use of a potential temperature is not necessary. It is sufficient to decide replacing the moist static energy h (notation not to be confused with the one for enthalpy used higher up)

$$h = C_p.T + g.z + L_{v/s}(T).q_v$$

by its 'moist entropic static energy' equivalent:

$$S_m \approx h - (L_{v/s}(T) - C_{pd}.T.\Lambda).q_t$$

The appearance of Λ in the new quantity and the proportionality of the 'innovation' to q_t make the bridge with all the previous considerations and let hope that the new quantity might lead to a more realistic diagnostic of moist unstable atmospheric conditions in the PBL.

Experiments were done in this sense with the ALARO version of ALADIN, looking at the behaviour of the shallow convection parameterisation, based on a comparison between moist static energy vertical gradients for the actual atmospheric state and at saturation (Geleyn, 1987). The results are indeed improved when moving, all other things equal, from h to S_m in the basic formulation, the progress being apparent in terms of both stability and consistency of the solution (Bašták-Ďurán, personal communication).

8. Conclusions and Outlook

Computing the diabatic forcing (and incorporating it correctly in the dynamical equations) undergoes a strong change of emphasis when reaching ‘NH scales’. The associated NWP-type impact is perhaps even more telling than the one of the change of the dynamical equations.

Our knowledge about what will give the forecast with most realism and reliability on the one hand, and least noise and costs on the other hand, is yet limited.

The clearer separation between ‘process description’ on one side and ‘code algorithmic superstructure’ on the other side offers a chance to see new paradigms emerging, more appropriate to the new situation.

Selecting those which will deserve a stable role will not be easy. It has been argued here that ‘consistency’ and some simple ‘transcription of the laws of thermodynamics’ might play a key role in this selection process.

The items of similar nature that we left aside for this note are linked to (i) the organisation of the ‘physical time-step’, (ii) the three dimensional aspects of diabatic forcing at NH scales (in particular for the lateral component of diffusion which ought to be treated as consistently as possible with respect to its vertical counterpart) and (iii) the link with stochastic modelling issues.

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