Multiscale Modeling Framework and Parameterization

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1. Introduction

The global numerical models used for weather prediction and climate simulation purport to determine area-averaged precipitation rates for each grid cell; naturally, some places in a grid cell get more precipitation that the predicted average, and others get less.

For a variety of reasons, discussed below, the grid-cell average precipitation rate and other grid-cell statistics are intrinsically uncertain (e.g., Hohenegger and Schär, 2007). Although it is true that uncertainties arise from deficiencies of the models, and from lack of precision in the initial conditions and boundary conditions, there is a more fundamental, irreducible component of uncertainty that comes from the probabilistic character of the predicted cloud parameters, which are merely ‘expected values’. As explained in the next section, this problem is more severe in the emerging high-resolution global models, which begin to resolve the mesoscale.

A second problem is that the classical assumptions about ‘quasi-equilibrium’ are never exact, and break down altogether when the time-scale for changes in the resolved-scale weather is comparable to or smaller than the convective adjustment time. Again, this problem is more severe in the emerging high-resolution global models, simply because convective systems with smaller spatial scales tend also to have shorter time scales.

Finally, as the resolution of our models increases, we must eventually confront the scale-dependence of physical processes themselves. With grid spacings of 200 km, the ‘convective mass flux’ is a highly relevant concept; with a grid spacing of 20 km, it is problematic for the larger clouds; with a grid spacing of 2 km, it is virtually meaningless. With the finer grid spacing, microphysical processes and turbulence become dominant.

These three problems have been recognized, to some extent, since the earliest days of cloud parameterization, but they are now becoming critical because of the increasingly urgent need for higher spatial resolution, for weather prediction and also to reveal regional detail in global climate simulations.

2. Quasi-equilibrium on the edge

2.1. Sample size and the required separation of spatial scales

The cumulus parameterizations that are used in virtually all forecast models and climate models are designed to represent the statistical effects of large numbers of clouds in a model grid column (e.g., Arakawa and Schubert, 1974; hereafter AS). Cumulus parameterizations entail the assumption that the convective clouds are in statistical equilibrium or ‘quasi-equilibrium’ with the time-varying large-scale weather conditions. In order for the concept of statistical equilibrium to be applicable, the number of clouds included in the sample must be sufficiently large. For example, AS wrote:

"Consider a horizontal area … large enough to contain an ensemble of cumulus clouds, but small enough to cover only a fraction of a large-scale disturbance. The existence of such an area is one of the basic assumptions of this paper."

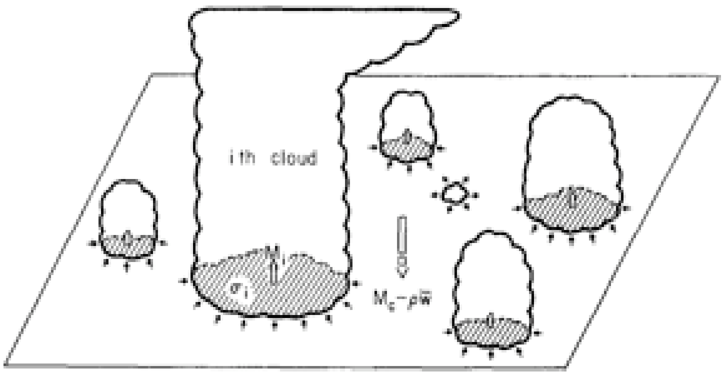

Figure 1 Sketch taken from Arakawa and Schubert (1974), showing an ensemble of cumulus clouds of various sizes, in a shared 'large-scale environment.'

See Fig. 1. Here we interpret the 'horizontal area' as the area of a grid column, although this is not necessary. The quote above poses two distinct requirements:

1. The number of clouds in the grid column must be sufficiently large to yield robust statistics, and
2. The grid column must be small enough to resolve a 'large-scale disturbance.'

The first requirement is closely related to the assumption that there is a clear separation of spatial scales between the convective clouds and the grid spacing. The meaning of the second requirement varies, depending on the range of grid resolutions considered; with grid spacings on the order of 20 km, 'large-scale' can really mean 'mesoscale.'

	Thermodynamics	**Cloud Parameterization**
Players	Molecules	Clouds
Volume	1 cubic cm	1 model grid column
Sample size	Trillions of molecules	Dozens to thousands of clouds
Simplifying assumptions	Point-like molecules; Inter-molecular collisions (usually negligible)	Small updraft area; Uniform environment; No direct interactions among clouds
Nonequilibrium effects	Brownian motion, etc.	TBD, possibly including mesoscale organization

Table 1: An analogy between cloud parameterization and thermodynamics. See the text for discussion.

The requirement of a sufficiently large sample of clouds suggests an analogy between cumulus parameterization and thermodynamics, which describes the statistical properties of a large number of molecules occupying a given volume, assumed to be small compared to the scale on which such statistics as temperature and pressure vary. Thinking about this analogy quickly raises concerns about the basic feasibility of cumulus parameterization (Table 1). To begin with, there is the alarming problem of sample size, already mentioned above. With typical large-scale model grid spacings, on the order of 100 km, the number of large cumulus clouds that can 'fit' simultaneously into a model grid column is only on the order of ten to a hundred. In fact, only a few thousand small cumulus clouds can fit into such a grid column. These cloud population sizes are miniscule compared to the many trillions of molecules occupying a single cubic centimeter under tropospheric conditions. The cloud populations are not large enough to yield robust statistics for such key quantities as heating and drying rates, i.e., a well defined statistical equilibrium does not exist. It follows that *heating and drying rates are parameterizable only with large error bars*.

Fig. 2 illustrates the problem of sample size for large clouds. The figure shows the instantaneous horizontal distribution of ice water path in a turbulence-resolving simulation of deep cumulus convection over the tropical oceans, under moderately disturbed conditions. Although the domain is about 200 km on a side, only a few large ice water patches are visible in the grid column.

The problem of sample size obviously becomes worse as the grid spacing decreases to a 20-50 km, a grid spacing already used in some global weather prediction models. This fundamental issue is a focus of the proposed research.

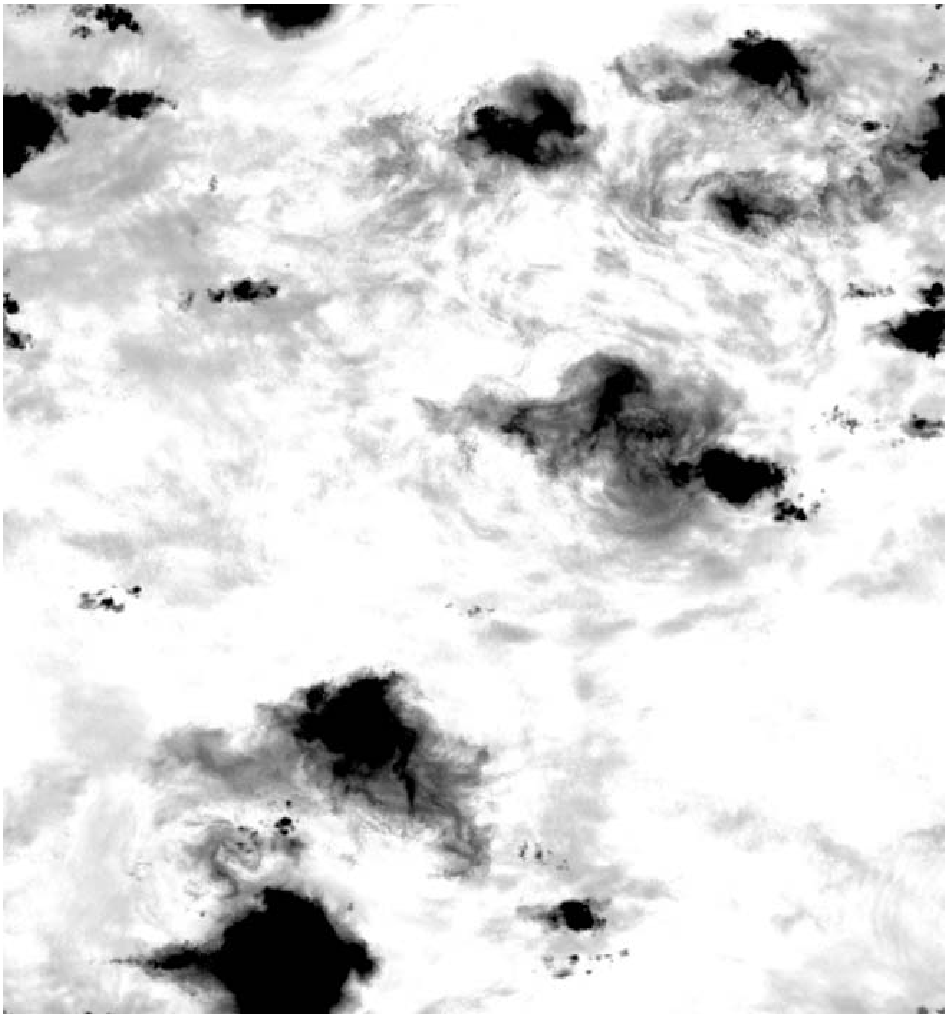

Figure 2: A snapshot of ice water path from a numerical simulation performed by M. Khairoutdinov. The domain size is 204.8 km square, and the horizontal grid spacing is 100 m. The specified forcing is loosely based on GATE Phase III. This image shows the state of the simulation almost 24 hours after initialization with random noise.

Existing cumulus parameterizations also involve simplifying assumptions that break down as a model's resolution increases. An important example is the assumption that the horizontal area covered by convective updrafts is small compared to the total area of the grid column. Obviously, this assumption must break down when the horizontal grid spacing is comparable to the size of a convective updraft.

2.2. Separation of time scales

A second fundamental issue has to do with time scales. AS wrote:

> *"When the time scale of the large-scale forcing, is sufficiently larger than the [convective] adjustment time, … the cumulus ensemble follows a sequence of quasi-equilibria with the current large-scale forcing. We call this … the quasi-equilibrium assumption…. The adjustment … will be toward an equilibrium state … characterized by … balance of the cloud and large-scale terms…"*

In other words, AS assumed that the convection can adjust the large-scale state on a time scale short compared to the time scale for the change of the large-scale state, just as an ensemble molecules can equilibrate on a time scale short compared to the time on which such statistics as temperature and moisture vary significantly. Various lines of evidence suggest that the 'adjustment time' for deep cumulus clouds is on the order of several hours, while that for smaller clouds is somewhat shorter.

When the grid-box-averaged weather changes on a time scale much longer than this cumulus adjustment time, the convection can stay close to statistical equilibrium, and the past history of the convection has little effect. The required separation of time scales is not adequate for the important case of the diurnal cycle, however, and may be marginal even for synoptic weather events.

In quasi-equilibrium, the large-scale 'forcing' and the sounding strongly determine the convective heating and drying rates, which adjust 'instantaneously' to the evolving large-scale weather. The instantaneous forcing determines the instantaneous convective heating and drying rates. The past history of the convection does not matter. The expected value of the convective response closely approximates the value in almost all realizations, simply because the variance is small compared to the mean. This quasi-equilibrium regime is illustrated in Fig. 3.

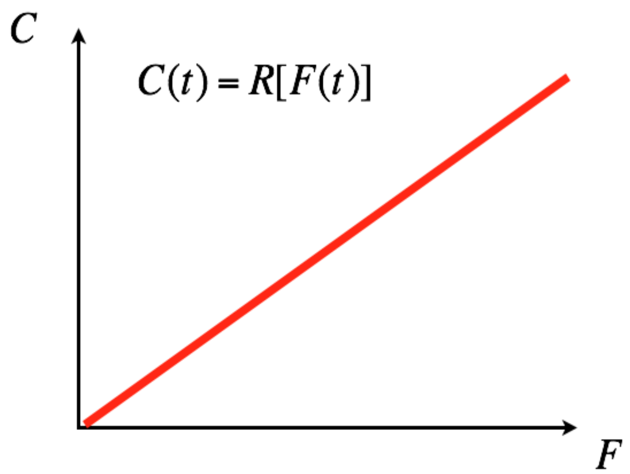

Figure 3: In the quasi-equilibrium regime, the convective response (vertical axis) is strongly determined by the time-varying large-scale forcing. The red line represents a simple and strong relationship between forcing and response.

When the large-scale weather evolves so rapidly that the convection cannot keep up, the past history of the convection affects the instantaneous convective heating and drying rates. Although the convection is out of equilibrium, the convective heating and drying rates are still deterministic. The *sequence* of large-scale forcings strongly determines the *sequence* of convective heating and drying rates. This deterministic, non-equilibrium regime is illustrated in Fig. 4.

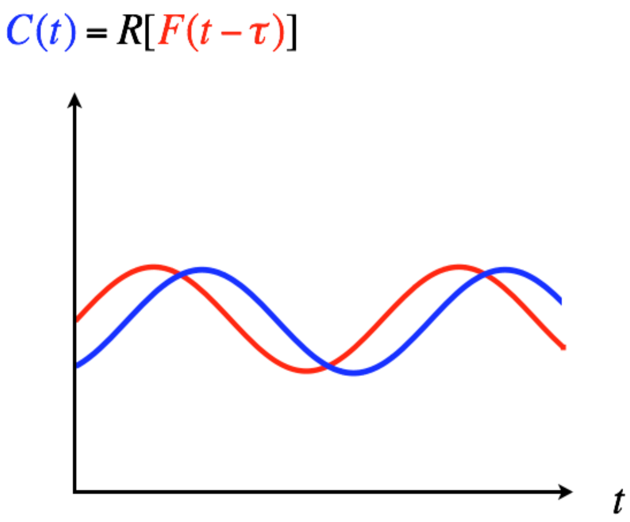

Figure 4: In the non-equilibrium regime, the convection (blue curve) cannot keep up with the time-varying large-scale forcing (red curve), because the convective adjustment time is too long.

With smaller grid spacings, in the rage 20-50 km, the time scale for important changes in the grid-averaged weather can be relatively short. For example, a mesoscale convective system such as a squall line can fill or nearly fill such grid cells, and typically passes through in an hour or so. For this reason, *the problem of the separation of time scales becomes more severe as the resolution of a model increases.*

When the sample size is too small to yield robust statistics, the convective heating and drying rates have a significant stochastic component, i.e., they are only partially predictable. Fig. 5 illustrates this non-deterministic regime. This stochastic or non-deterministic convection is expected when the grid columns are too small to contain a statistically adequate sample of clouds. The uncertainty in the convective heating and drying rates introduces an uncertainty in the larger scales. This can be true even when the convective adjustment time is small compared to the time-scale for the variation of the large-scale circulation. This type of uncertainty is characteristic of the cloud system (e.g., Hohenegger and Schär, 2007); it is part of the 'answer,' and cannot be eliminated by model refinements.

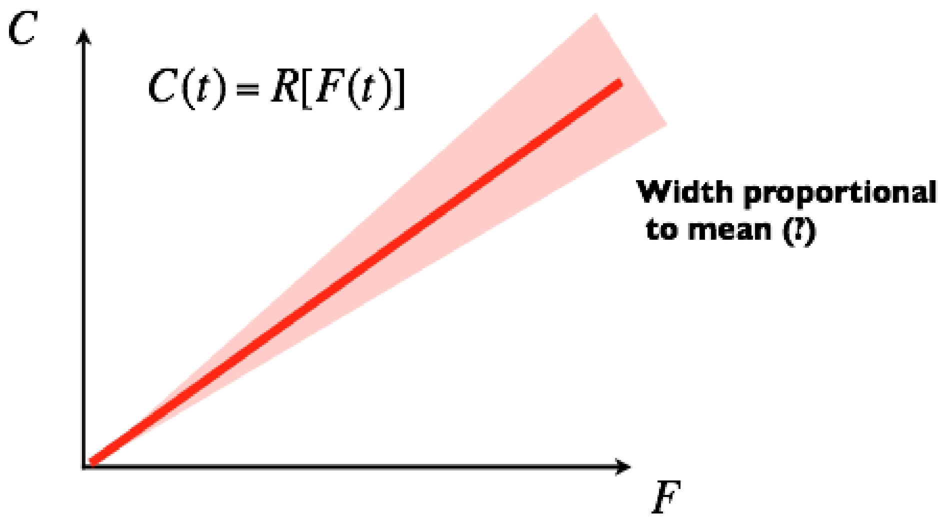

Figure 5: In the non-deterministic regime, the convective response is uncertain even when the convective adjustment time is short compared to the time scale for variation of the large-scale forcing. This occurs when the sample size is small, e.g., because the grid columns are small. In the figure, the pink 'fan' represents the uncertainty in the convective response. In this example, the uncertainty has been assumed to be proportional to the expected value of the convective response.

For the reasons discussed above, an equilibrium theory of cumulus convection should not be expected to explain in detail the observed interactions between convection and larger-scale weather systems. Nevertheless, the meteorological community has been single-mindedly pursuing such equilibrium theories for the past four decades. Only recently has non-equilibrium convection begun to attract attention (e.g., Xu et al., 1992; Pan and Randall, 1998; Buizza et al., 1999; Randall et al., 2003; Arakawa, 2004; Shutts and Palmer, 2007; Cohen and Craig, 2007; Plant and Craig, 2008).

2.3. An example with a cloud-resolving model

It is also possible, of course, to see a combination of the non-equilibrium and non-deterministic regimes. This is illustrated in the numerical simulations performed by Xu et al. (1992). They ran a two-dimensional cloud-resolving model with a horizontal domain size of 512 km. Although the width of the domain is large, the model's two-dimensionality sharply reduces the sample size, relative to a three-dimensional model with the same domain width. They specified a perfectly repeating time-varying large-scale forcing, with a period of 27 hours. This period is at most one order of magnitude longer than the adjustment time for the deep convective clouds.

Because the forcing is periodic in time, it is very easy to 'composite' or average together multiple periods to obtain the average convective response to the time-varying forcing. The results are shown in Fig. 6, in terms of the domain-averaged precipitation rate as a function of time. The convective response lags the forcing by several hours. The response includes a significant non-deterministic component, as measured by the standard deviation of the response for a given phase of the forcing. Some of the individual realizations shown in the left panel of Fig. 6 have instantaneous area-averaged precipitation rates that are twice the expected value as seen in the ensemble mean. These strong positive fluctuations show that extreme precipitation events can occur as non-deterministic fluctuations.

With a three-dimensional model, the sample size would increase for a given domain size, and so the 'scatter' in the composite plots would decrease, but the lag would remain unchanged. A larger domain would also increase the sample size, of course.

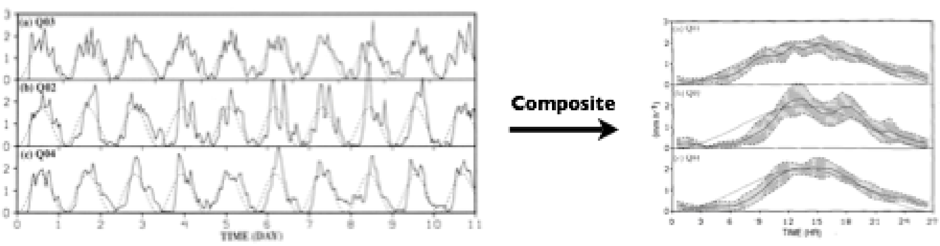

Figure 6: Numerical results obtained by Xu et al. (1992). The left panel shows the results from three numerical experiments, arranged vertically. The dashed curves show the periodic forcing. Although the three experiments have the same forcing, they differ in the specified vertical shear of the large-scale wind; see Xu et al. (1992) for details. The solid curves in the left panel show the simulated precipitation rate. The right panel shows the corresponding ensemble-mean results (solid curves) and the variations about the means (shading).

My student Todd Jones has performed simulations similar to those of Xu et al. (1992), but with a three-dimensional model. He used an idealized GATE-like forcing, with an imposed temporal periodicity. Fig. 7 shows how the cloud fraction and surface precipitation rate vary as functions of the strength of the forcing, in exploratory simulations with a 30-hour forcing period, and a domain width of 256 km. The left panel shows the surface precipitation rate, and the right panel shows the total cloud fraction. The surface precipitation rate lags the forcing by several hours. This can be interpreted as a consequence of the finite convective adjustment time. In contrast, the cloud amount actually leads the forcing, perhaps because the 'environmental subsidence' associated with the onset of deep convection favors the destruction of a portion of the pre-existing cloudiness. As seen in Fig. 8, a 'hysteresis' loop appears in the precipitation plot, because the precipitation rate increases somewhat slowly as the forcing increases with time.

2.4. Scale-dependence of heating and drying rates

A third fundamental issue involves the scale-dependence of the physical processes themselves. The area-averaged non-radiative 'apparent heat source' and 'apparent moisture sink' defined by Yanai et al. (1973) are given by

$$Q_1 - \bar{Q}_R = L\bar{C} - \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\overline{w's'}\right) - \frac{1}{\rho}\nabla_H \cdot \left(\rho\overline{\mathrm{v}'_H\, s'}\right),$$
$$Q_2 = -L\bar{C} - \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\overline{w'q'_v}\right) - \frac{1}{\rho}\nabla_H \cdot \left(\rho\overline{\mathrm{v}'_H\, q'_v}\right).$$

Here the overbars are very important; they represent an area average, which we interpret as a horizontal average over a grid cell. The expressions given above for Q_1 and Q_2 remain valid regardless of how large or small the grid cells are; the grid spacing can be 100 km, or 100 m. The leading terms on the right-hand sides represent the effects of condensation, the next terms represent vertical divergences of the convective 'eddy fluxes' of dry static energy and water vapor, respectively; and the last terms represent the horizontal divergences of the convective eddy fluxes, which are normally (and justifiably) neglected in large-scale models.

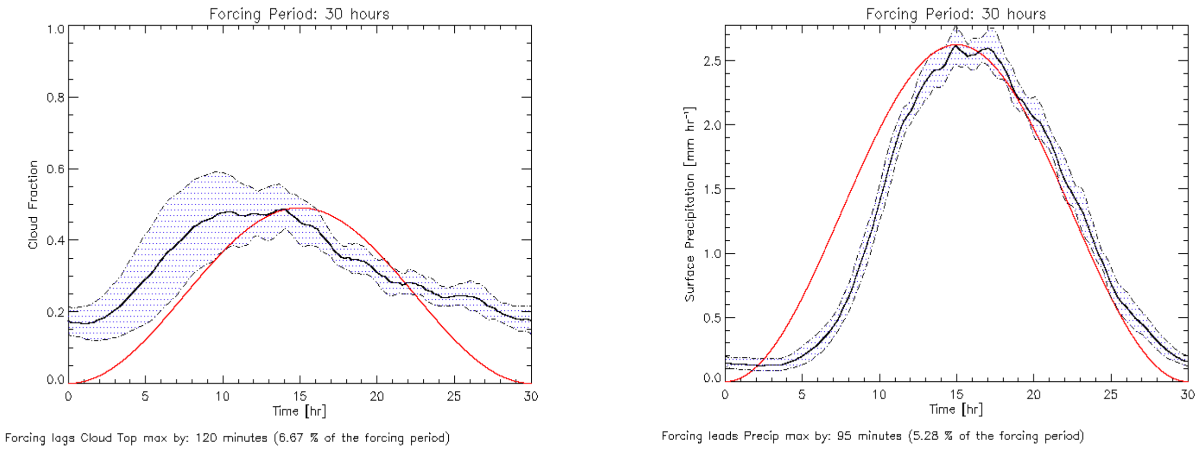

Figure 7: Results obtained in exploratory simulations using the three-dimensional cloud-resolving model of Jung and Arakawa (2008) with a 30-hour forcing period. The left panel shows the total cloud fraction (one minus the fraction of clear sky), and the right panel shows the surface precipitation rate . In each panel, the red curve shows the phase of the prescribed forcing. The calculations were performed by Todd Jones of Colorado State University.

As pointed out by Jung and Arakawa (2004), the roles and relative magnitudes of the various terms systematically change as the grid spacing becomes smaller:

- The vertical transport terms become less important. Later horizontal averaging does not alter this.
- The horizontal transport terms become more important locally. Horizontal averaging over sufficiently many grid columns renders them negligible, however.
- The phase-change terms become more important, and ultimately become dominant at high resolution.

Models intended for use with a coarse grid spacing, which includes all existing climate models, must use a parameterization that can represent the area-averaged effects of vertical eddy flux divergences due to cumulus clouds. ‘Mass-flux’ parameterizations (e.g., Arakawa and Schubert, 1974) are designed with this in mind. In contrast, models with much higher resolution appropriately focus on phase changes (as represented by microphysical processes), and place much less emphasis on the parameterization of eddy fluxes, which on fine scales are due only to turbulence.

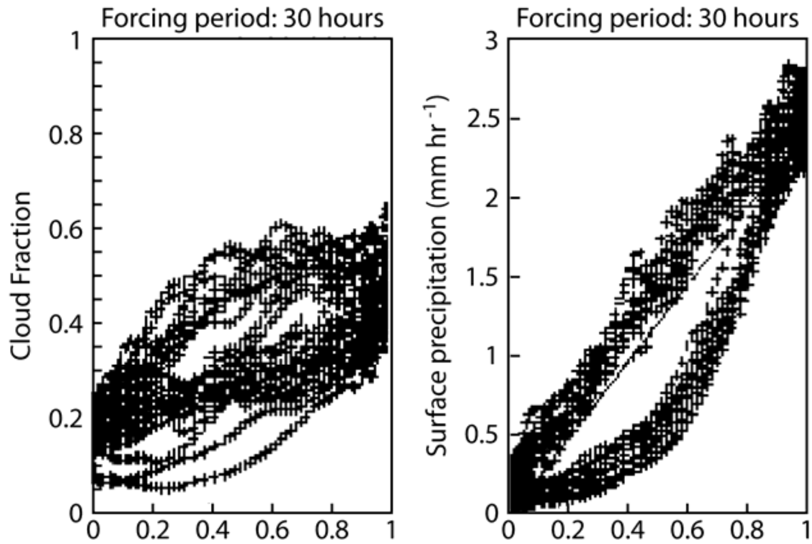

Figure 8: Simulated cloud fraction (left) and surface precipitation rate (right) as functions of forcing strength. These plots are based on the same results shown in Fig. 7.

2.5. Summary

Small sample sizes can lead to non-deterministic fluctuations of the convective heating and drying averaged over the grid column of a model. This problem becomes worse as a model’s resolution is increased, because the sample size diminishes.

Finite ‘adjustment times’ cause cloud systems to lag sufficiently rapid changes in the weather, so that quasi-equilibrium assumptions break down. This problem becomes worse as a model’s resolution is increased, because the time scales of the smaller-scale resolved circulations (e.g., mesoscale circulations) are shorter.

The mechanisms that produce ‘subgrid-scale’ heating and drying systematically change as the grid spacing is refined, for a given cloud regime. Vertical eddy flux divergences are paramount when the grid spacing is large, but microphysics ultimately dominates as the grid spacing is refined. For this reason, convection parameterizations that are physically realistic for use in coarse-resolution models are physically wrong when applied in higher-resolution models.

The three problems identified above are well known but rarely discussed. They are skeletons in our field’s closet. Model grid spacings are now increasing into the mesoscale range, so that all three problems become more severe. It is time to clean out the closet.

3. GCMs and CRMs

One solution to these three problems is to use a ‘cloud-resolving model’ (CRM), i.e., a model with a grid spacing that allows at least crude representation of individual large clouds. Such fine meshes have become routine in mesoscale models, but are only now becoming feasible in global models. To date, the only global cloud resolving model (GCRM) is that developed at the Frontier Research Center for Global Change, in

Japan (e.g., Tomita et al., 2005). Although numerical weather prediction with GCRMs may become possible within the coming decade, climate simulation with GCRMs is still several decades away, baring an unexpectedly dramatic advance in computing power (Wehner et al., 2008).

A less expensive alternative is the multi-scale modeling framework (MMF), in which a CRM is used as a ‘super-parameterization’ inside each grid column of an otherwise conventional GCM. This idea was first proposed by W. Grabowski (Grabowski,and P. K. Smolarkiewicz, 1999; Grabowski, 2001). It has been extended and applied by the Center for Multiscale Modeling of Atmospheric Processes (http://cmmap.colostate.edu/cmmap/index.html), as described in a series of papers (e.g., Khairoutdinov and Randall, 2001; Randall et al., 2003; Cole et al., 2005; DeMott et al., 2007; Khairoutdinov et al. 2005, 2008; Tao et al., 2008). The host GCM provides advective forcing to the CRM, as in a ‘single-column’ study based on field data, while the CRM provides heating and drying rates as feedback to the GCM.

In each GCM grid column of the MMF, the embedded CRM produces a realization of the convective activity that is consistent with the large-scale weather regime simulated by the GCM. The chaotic dynamics of the CRM ensures that the domain-averaged heating and drying have a stochastic component, as discussed in the preceding sections of this paper. The ‘super-parameterization’ represented by the CRM component of the MMF is, therefore, an example of a stochastic parameterization. It is expensive, but it has the major advantage that it is based directly on the equations of motion and provides a natural framework for the incorporation of microphysics, radiative transfer, and turbulence parameterizations. We can hope that in the future less expensive stochastic parameterizations will give results as good or better than those produced by MMFs, but for now this is merely a hope. We cannot even be sure that it is possible.

The MMF produces a robust simulation of the Madden-Julian Oscillation (MJO; e.g., Madden and Julian, 1971, 1994). A forthcoming paper by Benedict and Randall (2009) presents a detailed comparison of the MMF-simulated MJO with observations, The main conclusion is that the simulated MJO is quite realistic, except that it is stronger than observed.

A second forthcoming paper by Thayer-Calder and Randall (2009) analyzes particular physical processes that seem key to the MMF’s ability to generate an MJO. Emphasis is placed on the role of water vapor. The atmosphere is observed to gradually moisten during the weeks of ‘recharge’ (Bladé and Hartmann, 1993) preceding the heavy precipitation associated with the MJO. Grabowski (2003) has argued that this preconditioning of the humidity is a prerequisite for the growth of the numerous deep convective clouds that are characteristic of the disturbed phase of the MJO. On the other hand, Emanuel (1989) pointed out that convective downdrafts are inhibited in a very moist atmospheric column, because in the presence of high humidity the evaporation of falling rain cannot efficiently cool the air. Bony and Emanuel (2005) used a simple linear model to show that the inhibition of downdrafts by high humidity plays a role in the MJO. The results of Thayer-Calder and Randall (2009) support the idea that humidity preconditioning is an important process in the MJO, and are generally consistent with the theory of Bony and Emanuel (2005).

4. Concluding discussion

Cloud parameterizations represent relationships between the large-scale weather pattern and convective heating and drying rates. The relationships are by no means fully deterministic, nor are they purely diagnostic when the weather regime is rapidly changing. Emerging high-resolution global models must confront these issues, which are only now beginning to receive the attention that they require.

One approach to non-deterministic, non-equilibrium parameterization is the ‘super-parameterization,’ in which a CRM is embedded in each grid column of a large-scale model to create a Multiscale Modeling Framework. Tests of this approach have produced encouraging results, especially in simulations of the

Madden-Julian Oscillation. Efforts are under way to use MMF results to understand the processes that give rise to the MJO in nature, and to use this improved understanding to create more realistic conventional parameterizations.

5. Acknowledgments

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