Recent developments in the vertical discretization of the ECMWF model impacting on the stratosphere and tropopause

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Outline

- Finite-element discretization for the vertical
- Cubic spline interpolation in the vertical semi-Lagrangian advection
- Numerical instability during sudden stratospheric warming events at horizontal resolution of T_L511
- Increase in vertical resolution

Finite-element (FE) discretization for the vertical

- We use cubic B-splines as basis functions with compact support (finite elements).
- No staggering of variables used. All (including pressure) are held on the same set of levels (full levels). (Good for semi-Lagrangian advection.)
- Only non-local operations are evaluated in FE space, products of variables are evaluated in physical space. (Similar to spectral transform method in the horizontal.)
- In the semi-Lagrangian version of the ECMWF model, the only non-local operations in the vertical are integrations (no derivatives). Therefore, we have derived the FE form only for the integration operator.

FE scheme: Integral operator in finite-element form

$$F(x) = \int_0^x f(y)dy$$

Expanding f and F in terms of sets of linearly independent functions with compact support $\{e_i\}$ and $\{d_i\}$, respectively: $\sum_{i=1}^{M} C_i d_i(x) = \sum_{i=1}^{N} c_i \int_0^x e_i(y)dy$

Using the Galerkin method with $\{d_j\}$ as test functions

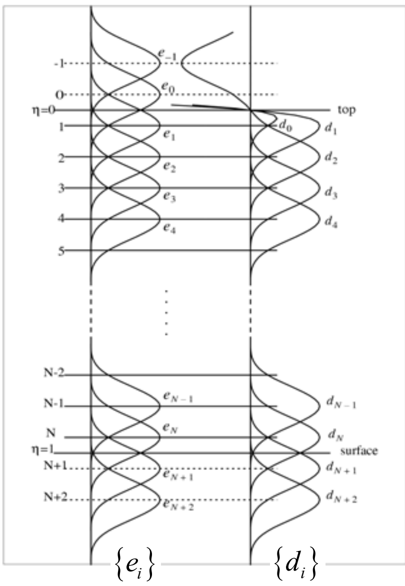
$$\sum_{i=1}^{M} C_i \int_0^1 d_j(x)d_i(x)dx = \sum_{i=1}^{N} c_i \int_0^1 [d_j(x)\int_0^x e_i(y)dy]dx, \quad j = 1,...,M$$

In matrix form: $\underline{\underline{A}}\,\underline{C} = \underline{\underline{B}}\,\underline{c} \quad \Leftrightarrow \quad \underline{C} = \underline{\underline{A}}^{-1}\underline{\underline{B}}\,\underline{c}$ (integral in FE space)

Incorporating the transformation to finite-element space and back into the Intergal operator, i.e. $\underline{c} = \underline{\underline{S}}^{-1}\underline{f} \quad \& \quad \underline{F} = \underline{\underline{\tilde{S}}}\,\underline{C}$

$$\Rightarrow \boxed{\underline{F} = \underline{\underline{\tilde{S}}}\,\underline{\underline{A}}^{-1}\underline{\underline{B}}\,\underline{\underline{S}}^{-1}\underline{f}}$$

FE scheme: Cubic B-splines as basis functions

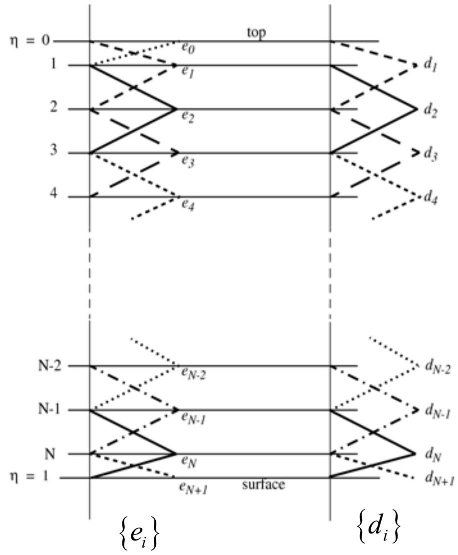


No staggering of basis set $\{d_i\}$ with respect to set $\{e_i\}$ (good for semi-Lagrangian adv.)

Condition F(0)=0 enforced by incorporation into basis functions, i.e. $d_i(0)=0$ for all i.
Basis functions d_0, d_1 & d_2 computed by linear combination of e_{-1} with e_0, e_1 & e_2, respectively.

Not restricted to a regular spacing of nodes.

FE scheme: Hat-functions (linear splines) as basis functions



FE scheme: Accuracy

Test: numerical integration of $\sin(6\pi x),\ x \in [0,1]$
for different resolutions with N equidistantly spaced nodes

Reference=analytical integral I_A. Error =max$\{(I_N-I_A)/I_A\}$ in %

	FD scheme	Linear FE	Cubic FE	Cubic collocation
N=60	0.82e+0	0.14e-2	0.90e-8	0.14e-2
N=120	0.21e+0	0.85e-4	0.31e-10	0.85e-4
estim. order	2	4	8	4

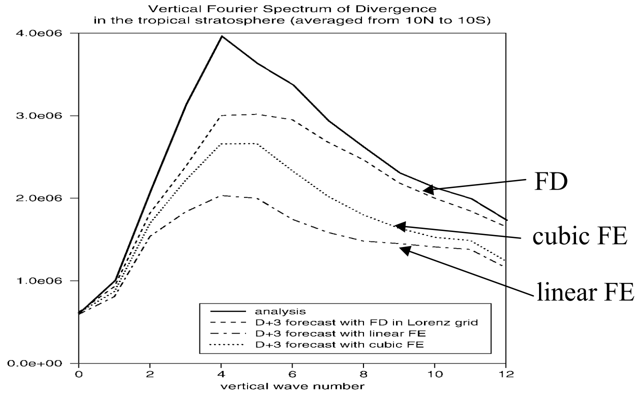
On nodes $O(h^{2(k+1)})$ where k is the degree of the basis functions and h the distance between nodes.

Superconvergence

Benefits from the FE scheme

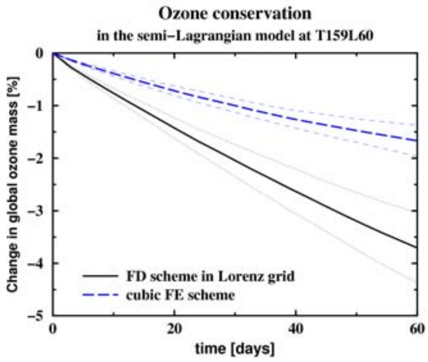
- **FE scheme improves the treatment of the gravity wave terms and dampens the computational (zigzag) mode in the vertical present in finite-difference schemes with no staggering of winds and temperature (Lorenz grid).**

=> Reduces the amplitude of grid-wave noise in the stratosphere.

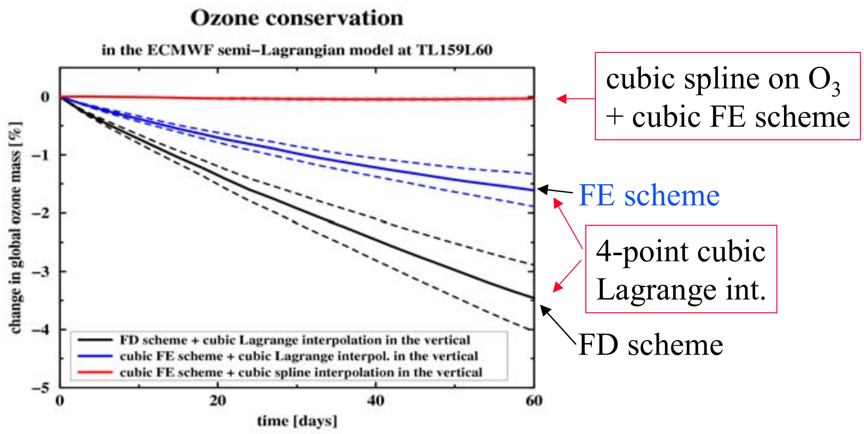


Benefits from the FE scheme (2)

Improved vertical integration of the continuity equation leads to a more accurate vertical velocity for semi-Lagrangian advection.
=> **improved tracer conservation**

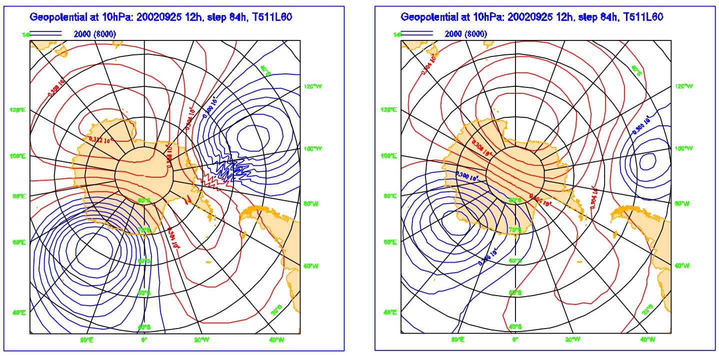


Cubic spline interpolation for the vertical in the semi-Lagrangian advection



The vertical interpolation error with the 4-point cubic Lagrange interpolation is responsible for quite a substantial part of the non-conservation of tracers with the semi-Lagrangian advection scheme. Better interpolation (cubic spline) => improved conservation.

Numerical noise during sudden stratospheric warming (1)

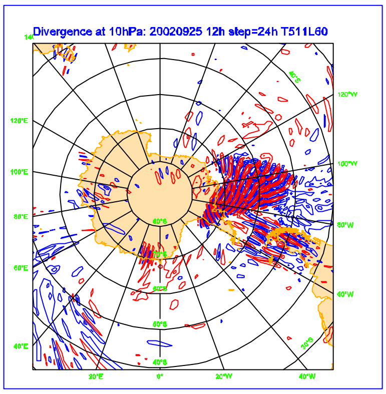


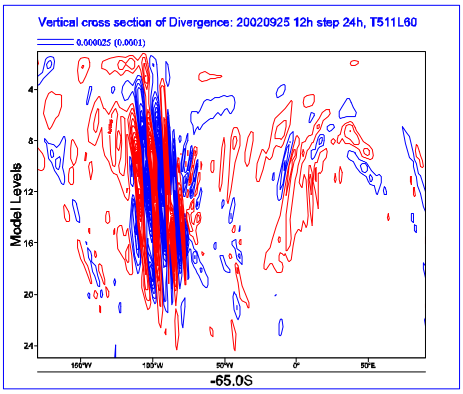
Noise appears only in integrations at high horizontal resolution (T_L511).

It is highly predictable (up to 8 days ahead) suggesting that it is linked to a specific well-predicted feature of the large-scale flow.

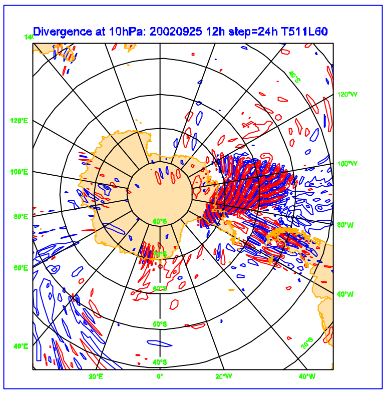
This computational instability does not make the forecasts fail. The noise disappears again when flow pattern changes back to more normal conditions.

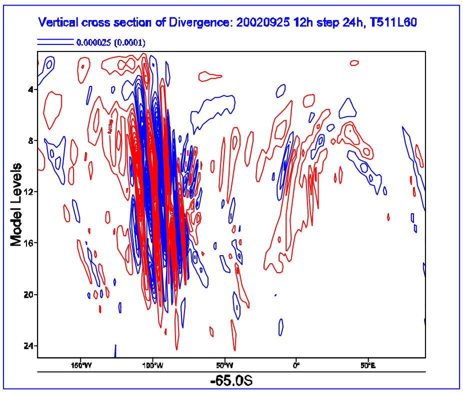
Numerical noise during sudden stratospheric warming (2)





Numerical noise during sudden stratospheric warming (2)



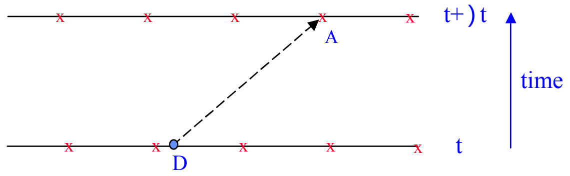


Noise during sudden stratospheric warming (3)

Vertical trajectory calculation for semi-Lagrangian advection:

$$\eta_A(t+\Delta t) = \eta_D(t) + \Delta t \frac{\dot{\eta}_A(t) + \left[2\dot{\eta}(t) - \dot{\eta}(t-\Delta t)\right]_D}{2}$$

where η denotes the vertical and $\dot{\eta}$ the vertical velocity.
'*A*' and '*D*' stand for arrival and departure point, respectively.

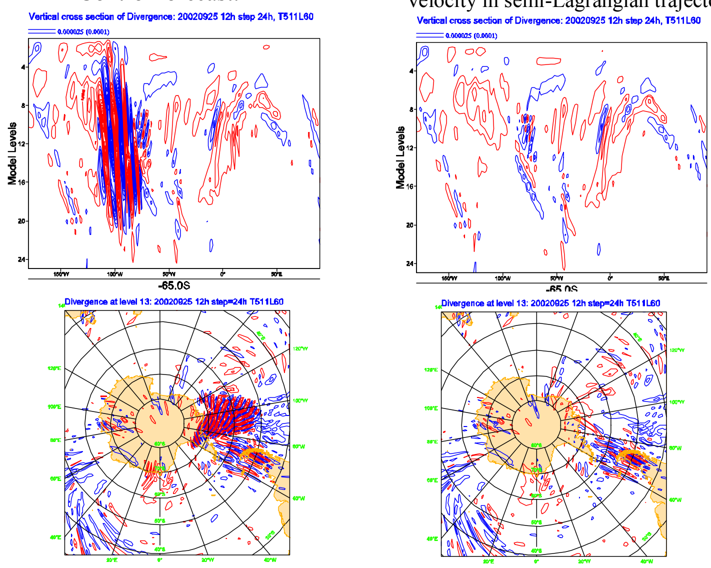


The formation of this noise can be suppressed if the vertical velocity used in the semi-Lagrangian trajectory computation is smoothed in the horizontal by applying a least square fit through four surrounding points instead of just linear interpolation between two points.
This is done only for the vertical velocity used in vertical trajectory calculation. All other variables, including the vertical velocity 'seen' by the physical parametrizations and the thermodynamic equation, are not being smoothed!

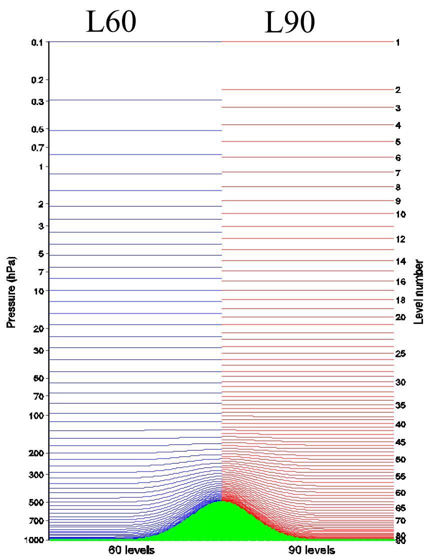
Numerical noise during sudden stratospheric warming (4)

Control forecast:

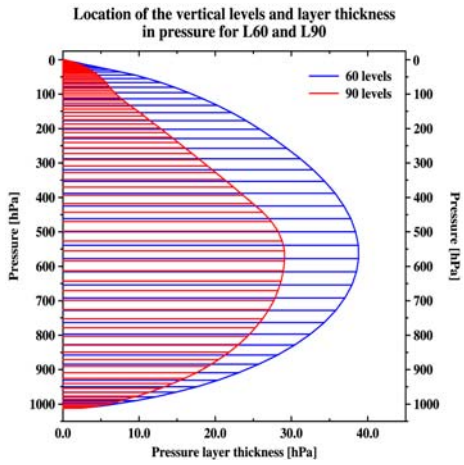
Forecast with horizontally smoothed vertical velocity in semi-Lagrangian trajectory:



Increase in vertical resolution
from 60 to 90 levels

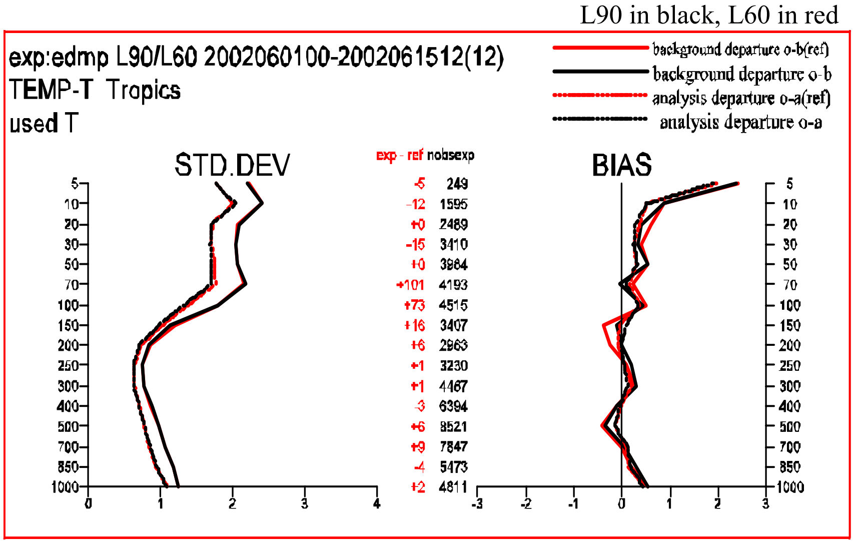


Vertical resolution doubled around tropical tropopause.

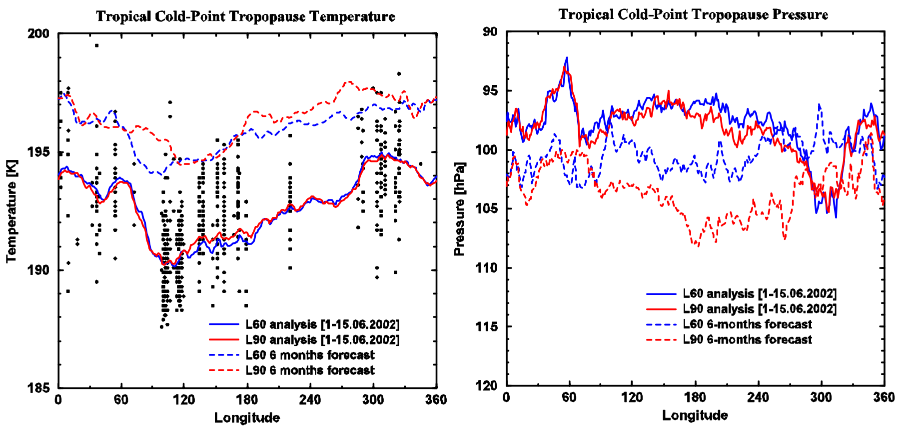


L90:
Improved fit of the analysis and background to radiosonde temperatures in the tropics with 90-level model

L90 in black, L60 in red

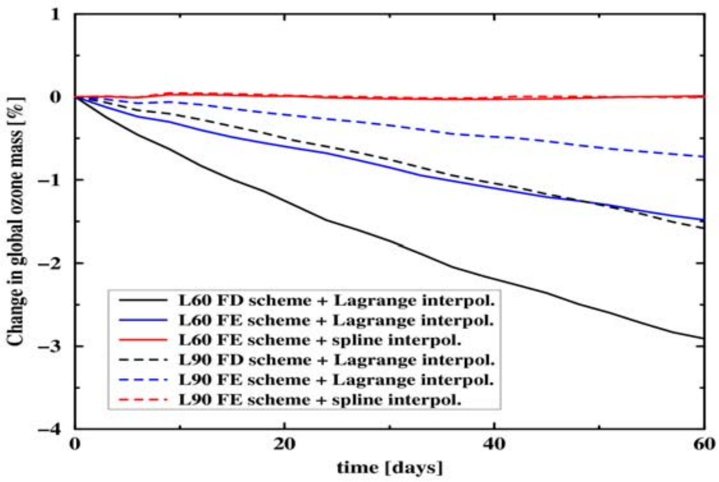


Tropical Cold-Point Tropopause with L60 and L90

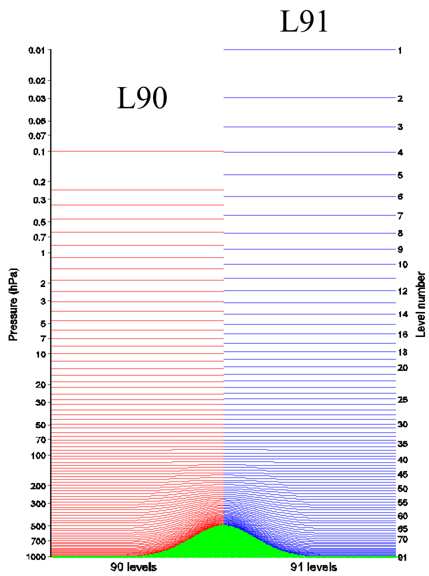


Black dots represent radiosonde temperature measurements.
Averaged over the deep tropics [10S to 10N].
Analyses averaged in time from 20020601 to 20020615, forecasts averaged over whole month of June.

L90: Positive impact on ozone conservation

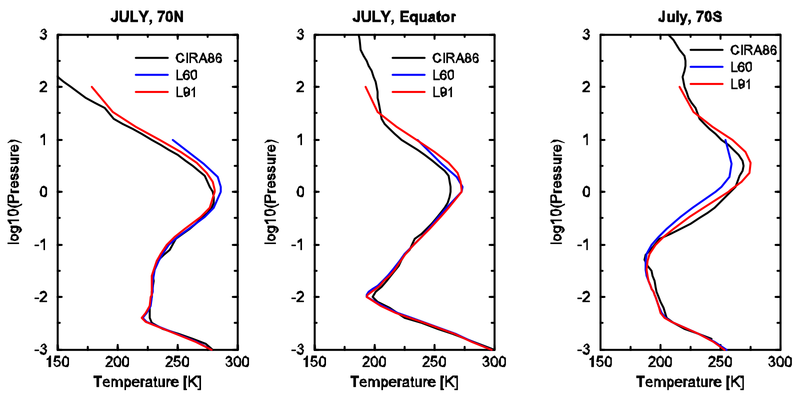


L91: Model top raised from 0.1hPa to 0.01hPa

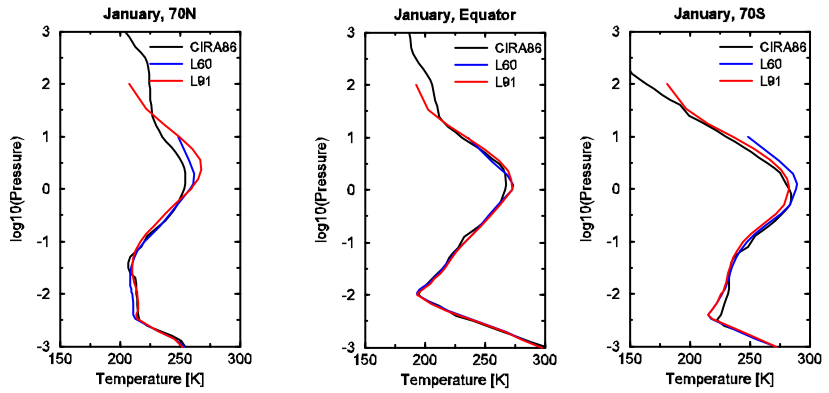


L91: Comparison with CIRA86 Climatology for July

Average July temperatures from climate runs with 91-level and 60-level model compared to CIRA86 climatology. Model with higher top (L91) performs better.



L91: Comparison with CIRA86 Climatology for January



Summary

- A finite-element discretization of the vertical was presented
  - Improves the stratosphere by reducing the vertical grid-wave noise present in the finite-difference scheme in Lorenz staggering.
  - Improves conservation of tracers due to improved vertical transport in stratosphere.
- Cubic spline interpolation in the vertical semi-Lagrangian advection
  - Improves conservation of ozone in semi-Lagrangian scheme through a reduction in interpolation error in the lower stratosphere and near the tropopause
- A numerical instability during stratospheric warmings was discussed and a solution presented.
- Results from tests with increased vertical resolution (L90 and L91) were presented.
  - Largest resolution increase is near the tropopause in both L90 and L91. In L91 the top is raised from 0.1hPa (in L60 and L90) to 0.01hPa.

Future work

- **Based on the L90 or L91 model version we will try to**
  - **improve vertical transport in the stratosphere.**
    - ➔ benefit for ozone assimilation and interactive ozone with radiation
  - **reduce large model errors near the stratopause by implementing a non-orographic gravity-wave drag parametrization.**
    - ➔ less problems with assimilation of satellite data
  - **improve upper boundary condition.**

References:

More detail about the finite-element scheme can be found in

Untch, A. and M. Hortal (2003): A finite-element scheme for the vertical discretization in the semi-Lagrangian version of the ECMWF forecast model. ECMWF Tech. Memo. 382.

For additional information about the numerical noise in the stratosphere during sudden warmings see

Simmons, A., M. Hortal, A. Untch and S. Uppala (2003): Breakdown of the stratospheric winter polar vortex. ECMWF newsletter No 96, Winter 2002-2003, PP 2-10.

Hortal, M., A. Untch (2003): A new interpolation for the vertical computation of the semi-Lagrangian trajectory. ECMWF research department memorandum, R60.5/MH/0415