Recent developments in the vertical discretization of the ECMWF model with impact on the stratosphere and tropopause

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with help from many colleagues at ECMWF

Outline

- Finite-element discretization for the vertical
- Cubic spline interpolation in the vertical semi-Lagrangian advection
- Numerical instability during sudden stratospheric warming events at T_L511
- Increase in vertical resolution (60 levels → 90 levels)

Finite-element (FE) discretization for the vertical

- We use cubic B-splines as basis functions with compact support (finite elements).
- No staggering of variables used. All (including pressure) are held on the same set of levels (full levels). (Good for semi-Lagrangian advection.)
- Only non-local operations are evaluated in FE space, products of variables are evaluated in physical space. (Similar to spectral transform method in the horizontal.)
- In the semi-Lagrangian version of the ECMWF model, the only non-local operations in the vertical are integrations (no derivatives). Therefore, we have derived the FE form only for the integration operator.

FE scheme: Integral operator in finite-element form

$$F(x) = \int_0^x f(y)\,dy$$

Expanding f and F in terms of sets of linearly independent functions with compact support $\{e_i\}$ and $\{d_i\}$, respectively:

$$\sum_{i=1}^{M} C_i d_i(x) = \sum_{i=1}^{N} c_i \int_0^x e_i(y)\,dy$$

Using the Galerkin method with $\{d_j\}$ as test functions

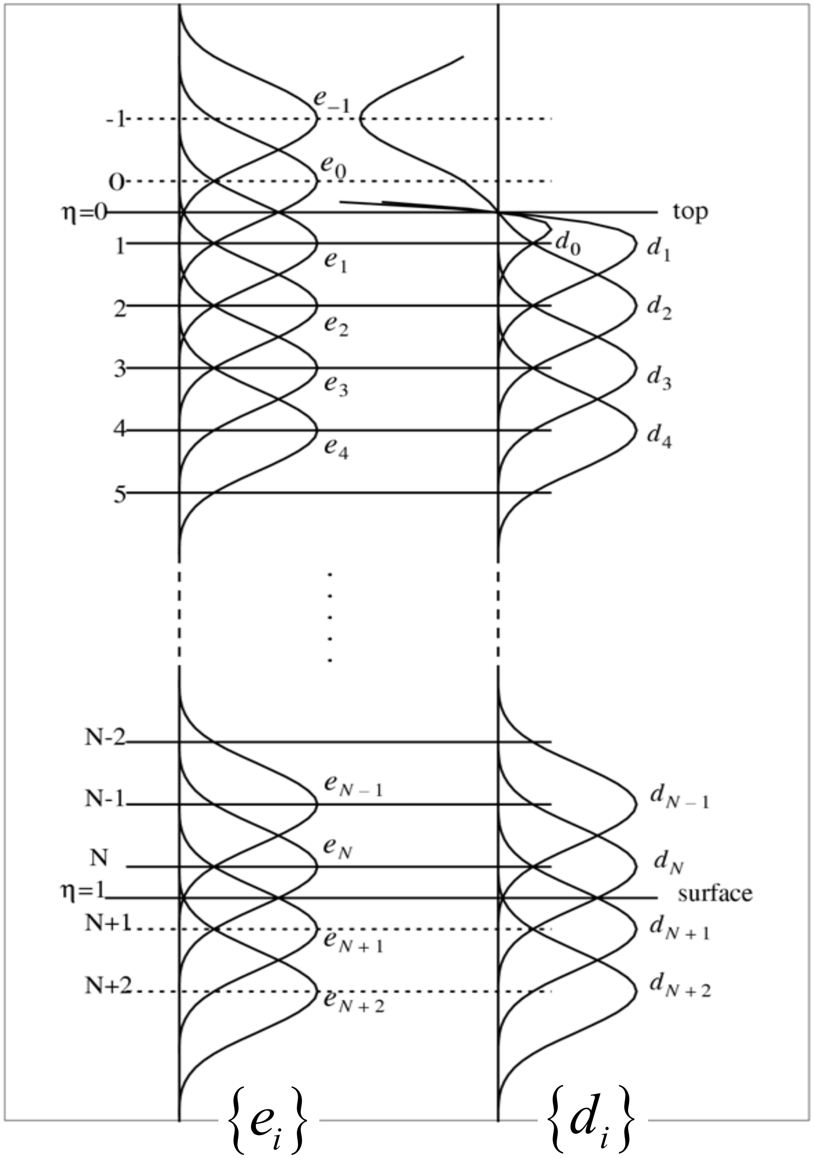
$$\sum_{i=1}^{M} C_i \int_0^1 d_j(x) d_i(x)\,dx = \sum_{i=1}^{N} c_i \int_0^1 [d_j(x) \int_0^x e_i(y)\,dy]\,dx, \quad j = 1,\ldots,M$$

In matrix form: $\underline{\underline{A}}\,\underline{C} = \underline{\underline{B}}\,\underline{c} \quad \Leftrightarrow \quad \underline{C} = \underline{\underline{A}}^{-1}\underline{\underline{B}}\,\underline{c}$ (integral in FE space)

Incorporating the transformation to finite-element space and back into the Intergal operator, i.e. $\underline{c} = \underline{\underline{S}}^{-1}\underline{f} \quad \& \quad \underline{F} = \underline{\underline{\tilde{S}}}\,\underline{C}$

$$\Rightarrow \quad \underline{F} = \underline{\underline{\tilde{S}}}\,\underline{\underline{A}}^{-1}\underline{\underline{B}}\,\underline{\underline{S}}^{-1}\underline{f}$$

FE scheme: Cubic B-splines as basis functions



No staggering of basis set $\{d_i\}$ with respect to set $\{e_i\}$ (good for semi-Lagrangian adv.)

Condition F(0)=0 enforced by incorporation into basis functions, i.e. $d_i(0)=0$ for all i.
Basis functions d_0, d_1 & d_2 computed by linear combination of e_{-1} with e_0, e_1 & e_2, respectively.

Not restricted to regular spacing of nodes.

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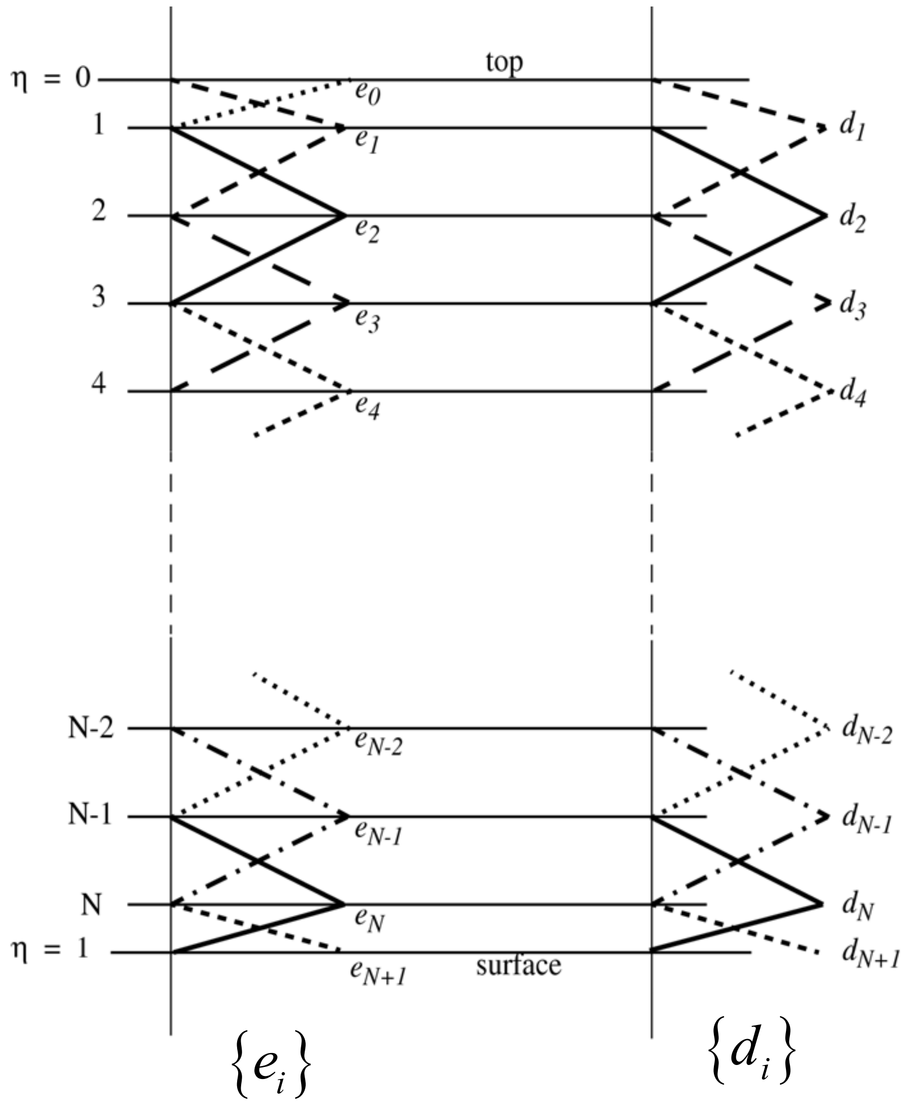
FE scheme: Hat-functions (linear splines) as basis functions



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FE scheme: Accuracy

Test: numerical integration of $\sin(6\pi x),\ x \in [0,1]$

for different resolutions with N equidistantly spaced nodes

Reference=analytical integral I_A. Error =max$\{(I_N-I_A)/I_A\}$ in %

	FD scheme	Linear FE	Cubic FE	Cubic collocation
N=60	0.82e+0	0.14e-2	0.90e-8	0.14e-2
N=120	0.21e+0	0.85e-4	0.31e-10	0.85e-4
estim. order	2	4	8	4

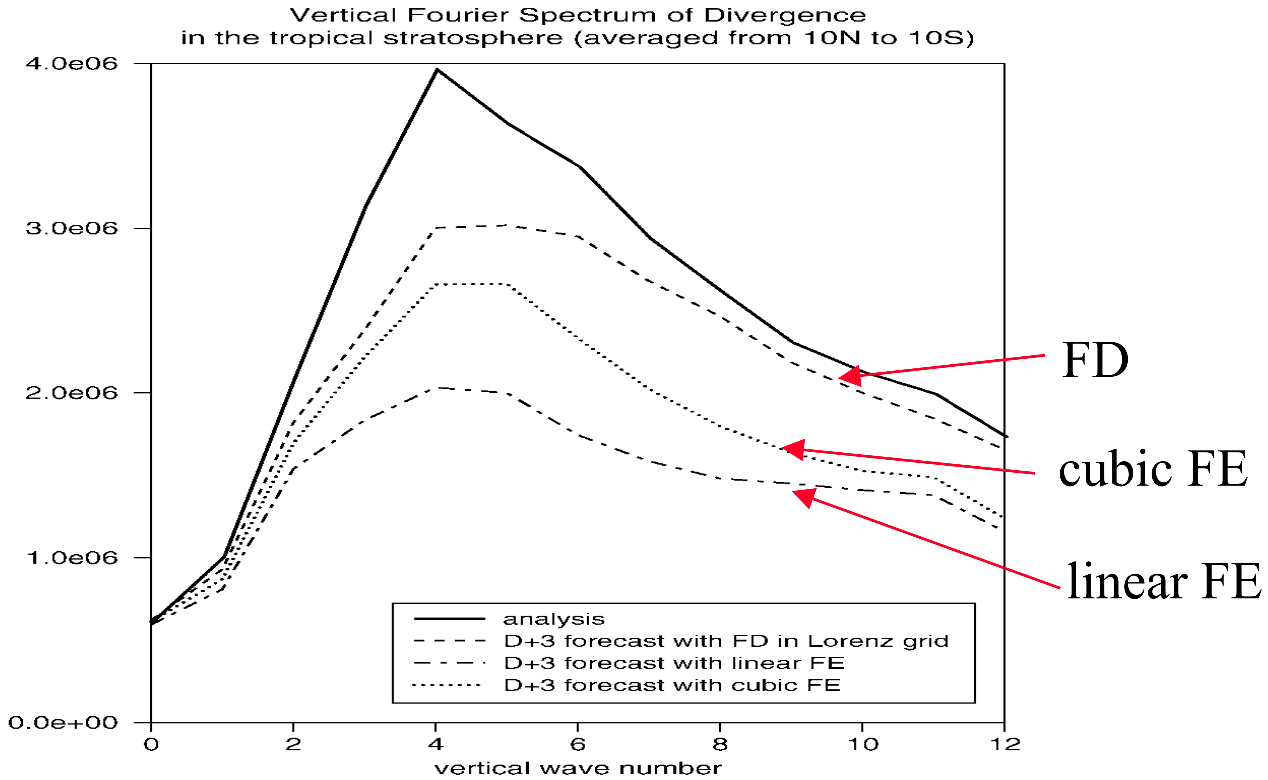
On nodes $O(h^{2(k+1)})$ where k is the degree of the basis functions and h the distance between nodes

Superconvergence

Benefits from the FE scheme

- **FE scheme improves the treatment of the gravity wave terms and dampens the computational (zigzag) mode in the vertical present in finite-difference schemes with no staggering of winds and temperature (Lorenz grid).**

=> Reduces the amplitude of grid-wave noise in the stratosphere.



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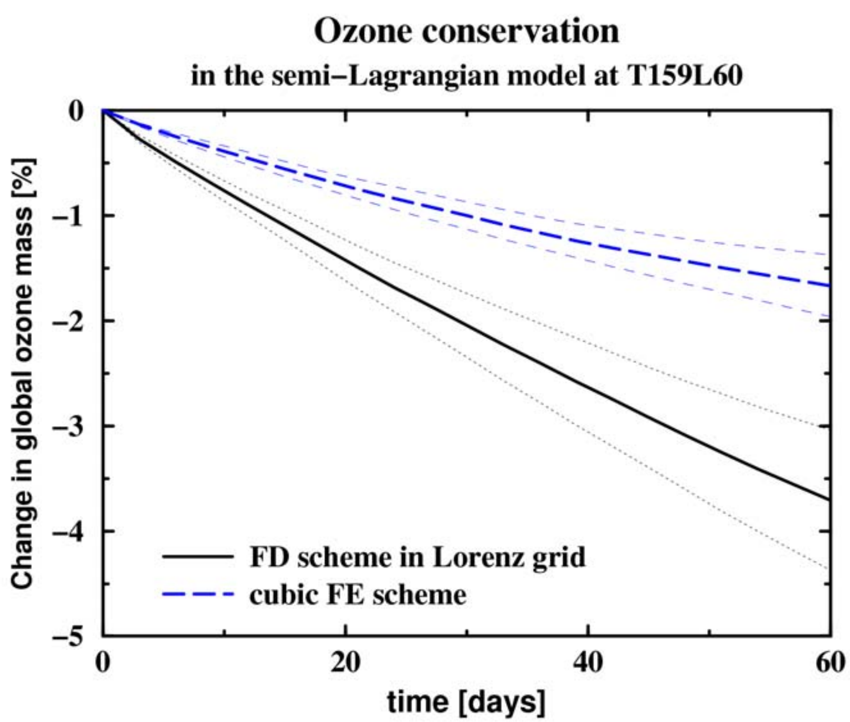
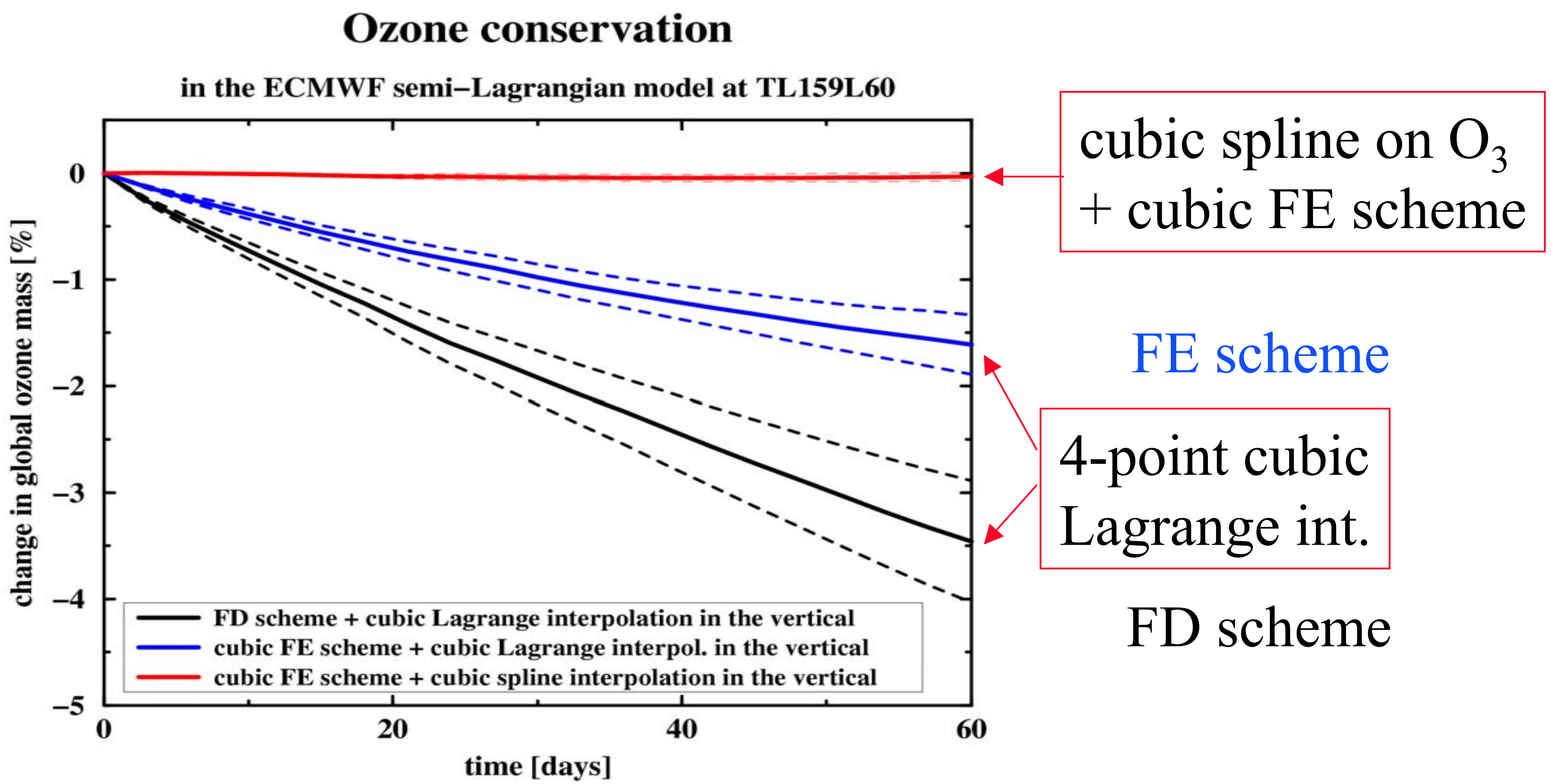
Benefits from the FE scheme (2)

Improved vertical integration of the continuity equation leads to a more accurate vertical velocity for semi-Lagrangian advection.
=> improved tracer conservation



Cubic spline interpolation for the vertical in the semi-Lagrangian advection



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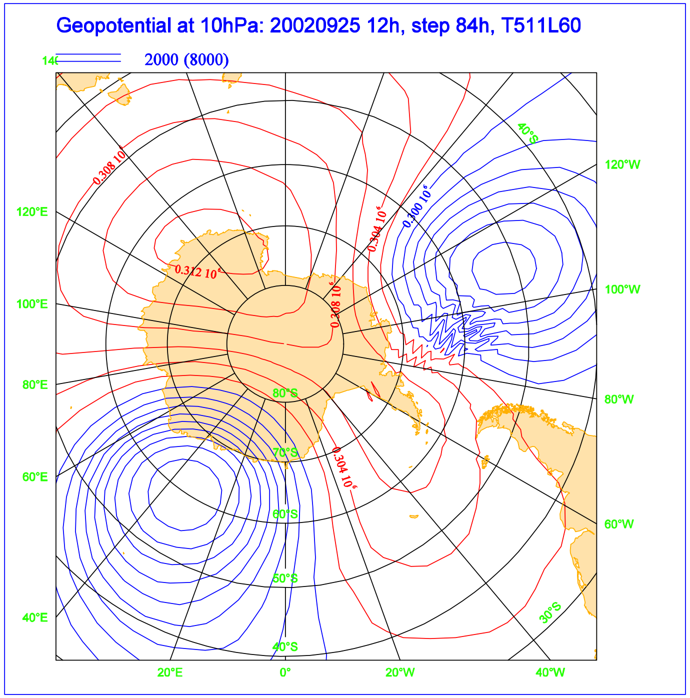
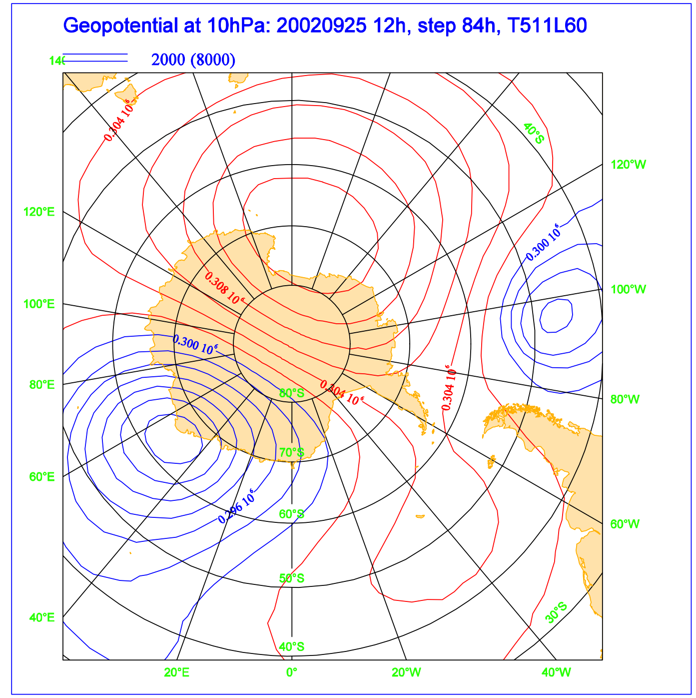
Numerical noise during sudden stratospheric warming (1)



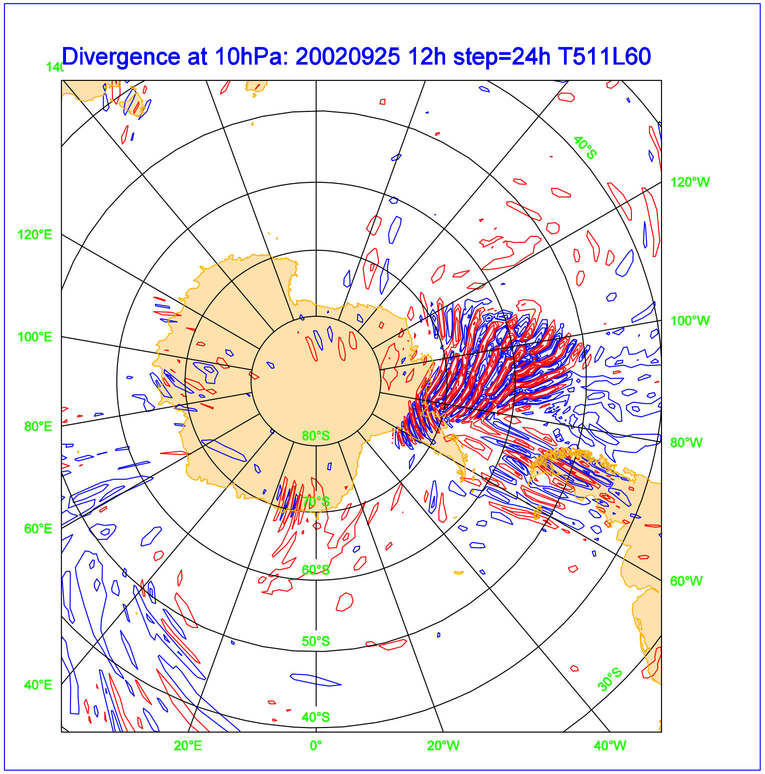


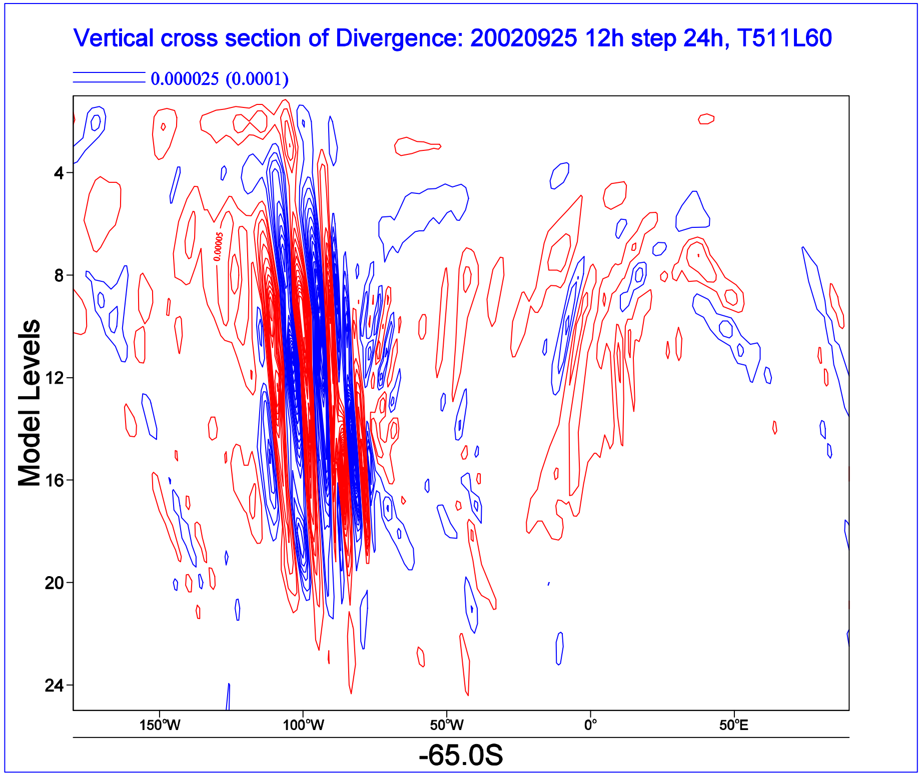
Noise appears only in integrations at high horizontal resolution (T_L511)

It is highly predictable (up to 8 days ahead) suggesting that it is linked to a specific well-predicted feature of the large-scale flow.

Forecasts don’t fail, noise disappears again when flow pattern changes

Numerical noise during sudden stratospheric warming (2)





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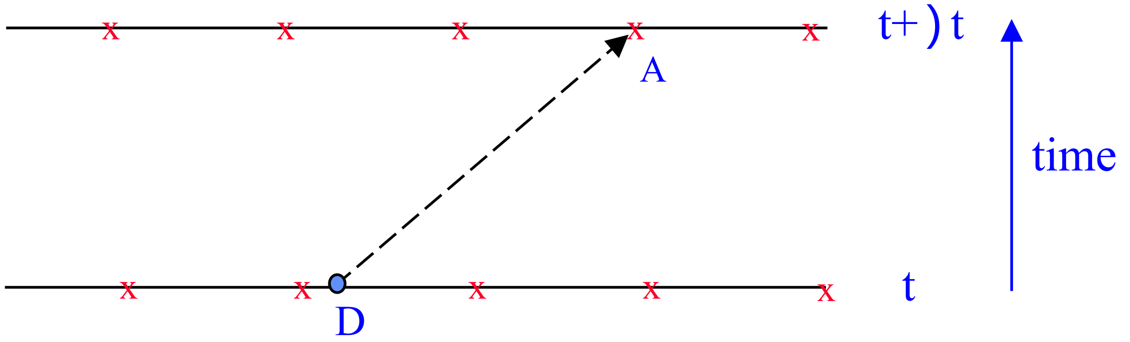
Noise during sudden stratospheric warming (3)

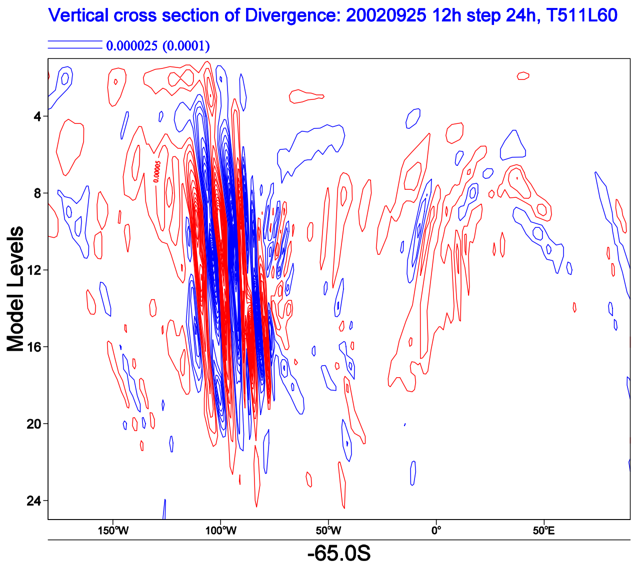
Vertical trajectory calculation for semi-Lagrangian advection:

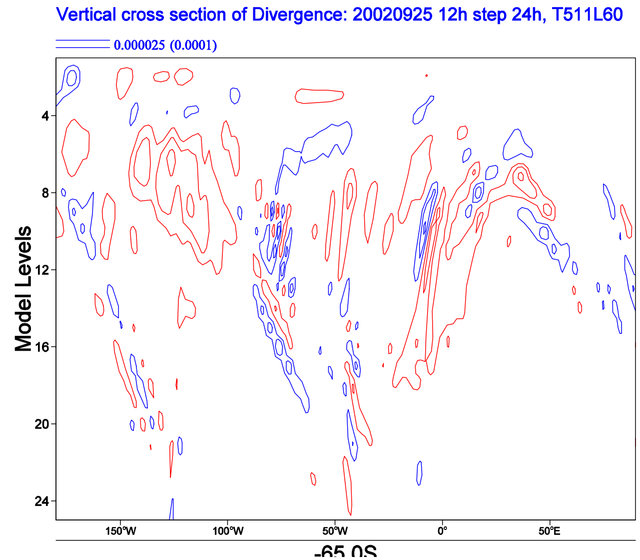
$$\eta_A(t+\Delta t) = \eta_D(t) + \Delta t \frac{\dot{\eta}_A(t) + \left[2\dot{\eta}(t) - \dot{\eta}(t-\Delta t)\right]_D}{2}$$

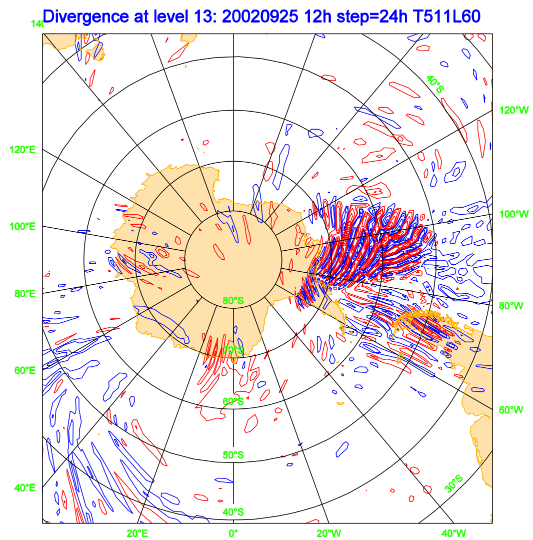


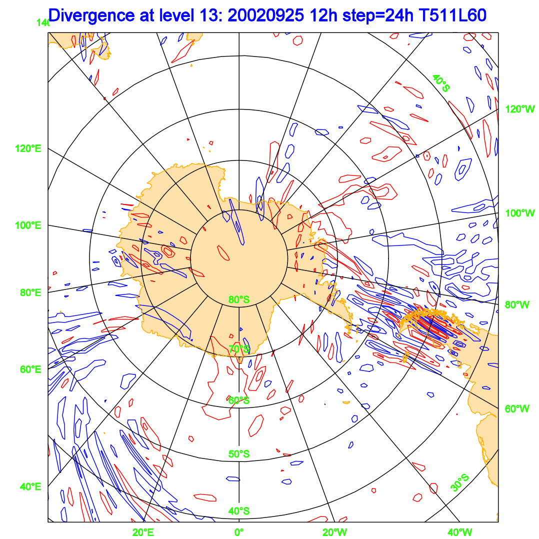
Smoothing of vertical velocity by least square fit through 4 surrounding points instead of just linear interpolation between 2 points. Done only for vertical velocity used in vertical trajectory calculation.

Numerical noise during sudden stratospheric warming (4)

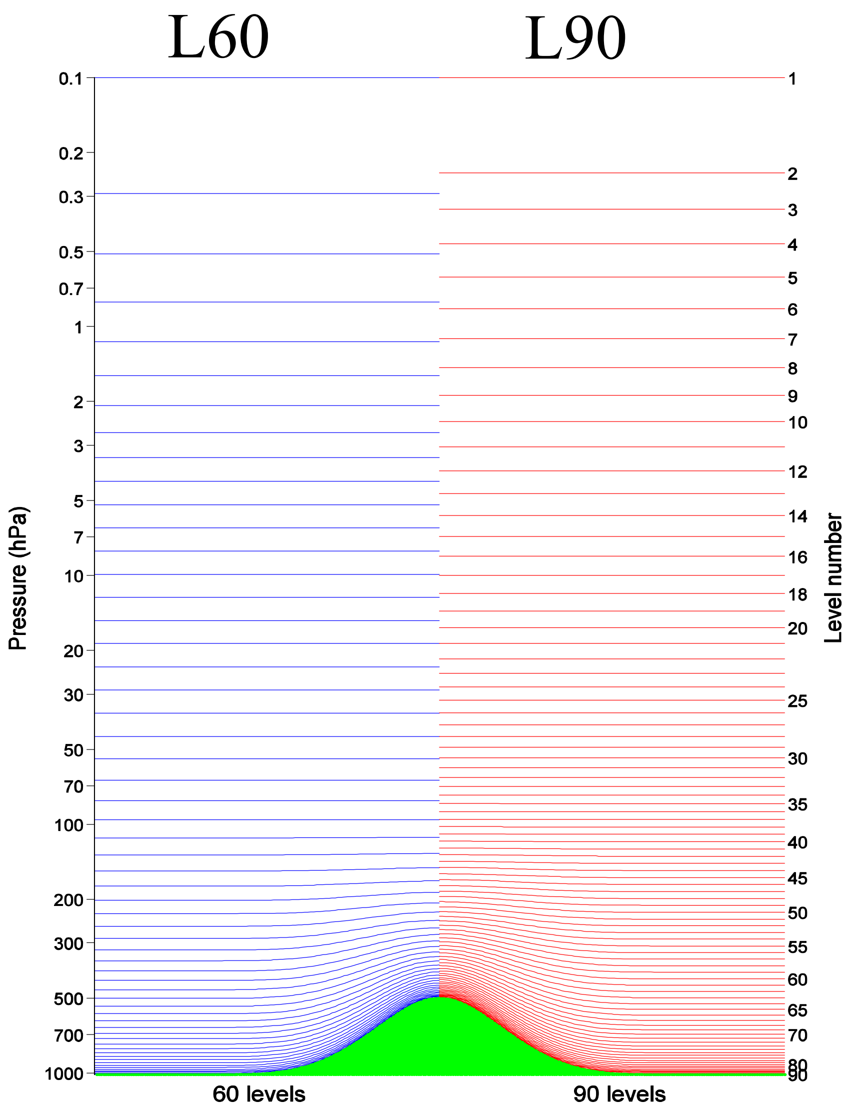


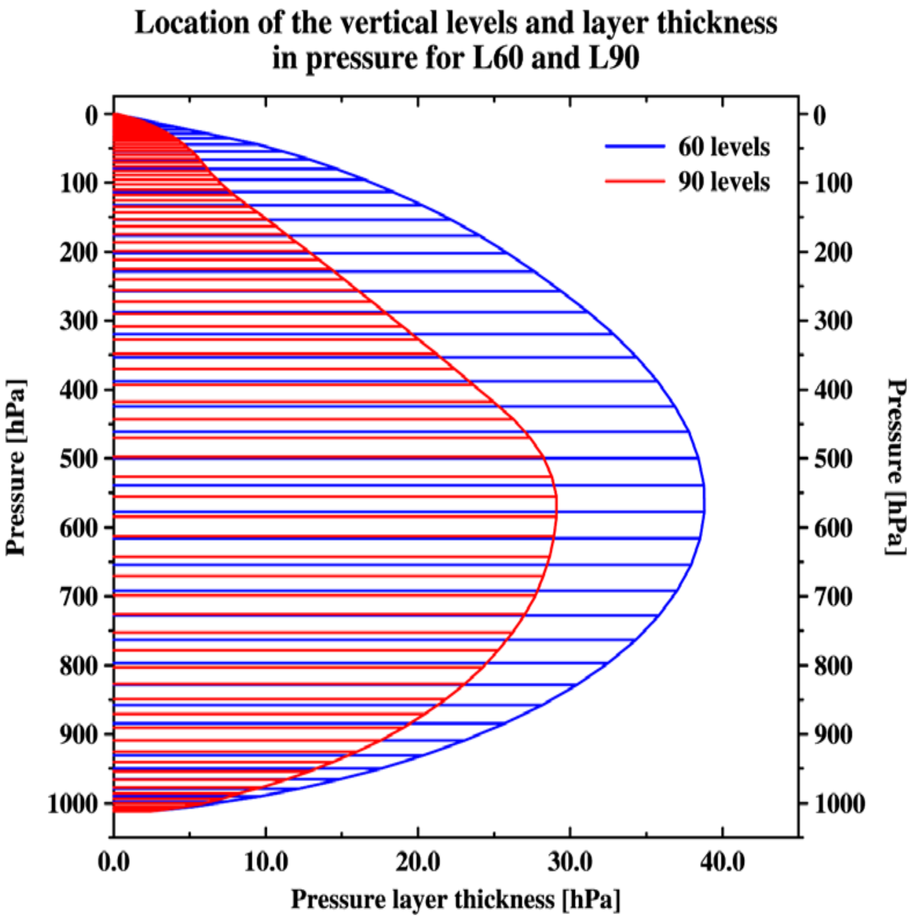






Increase in vertical resolution





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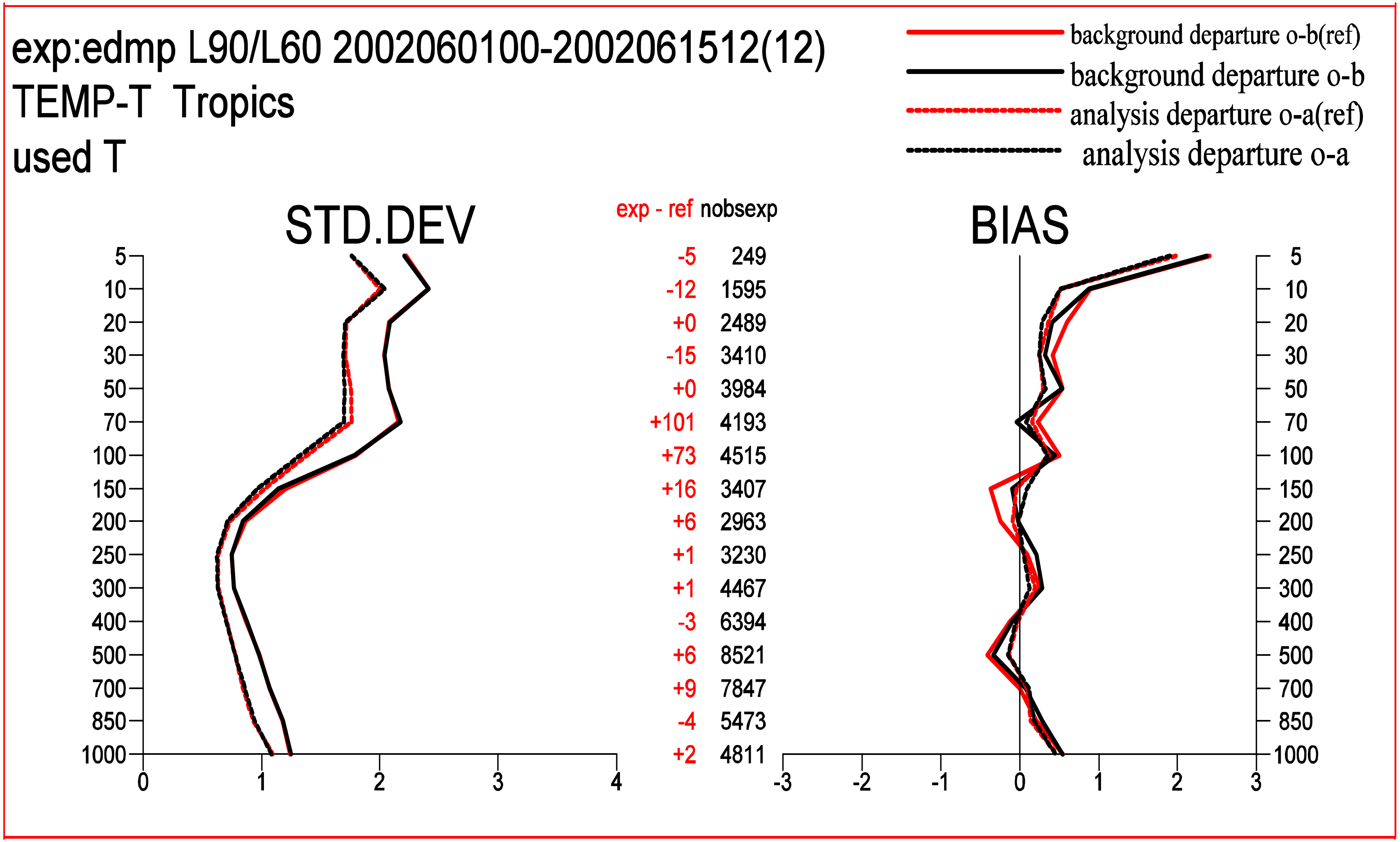
L90: Fit to Radiosonde Temperatures in the Analysis

L90 in black, L60 in red



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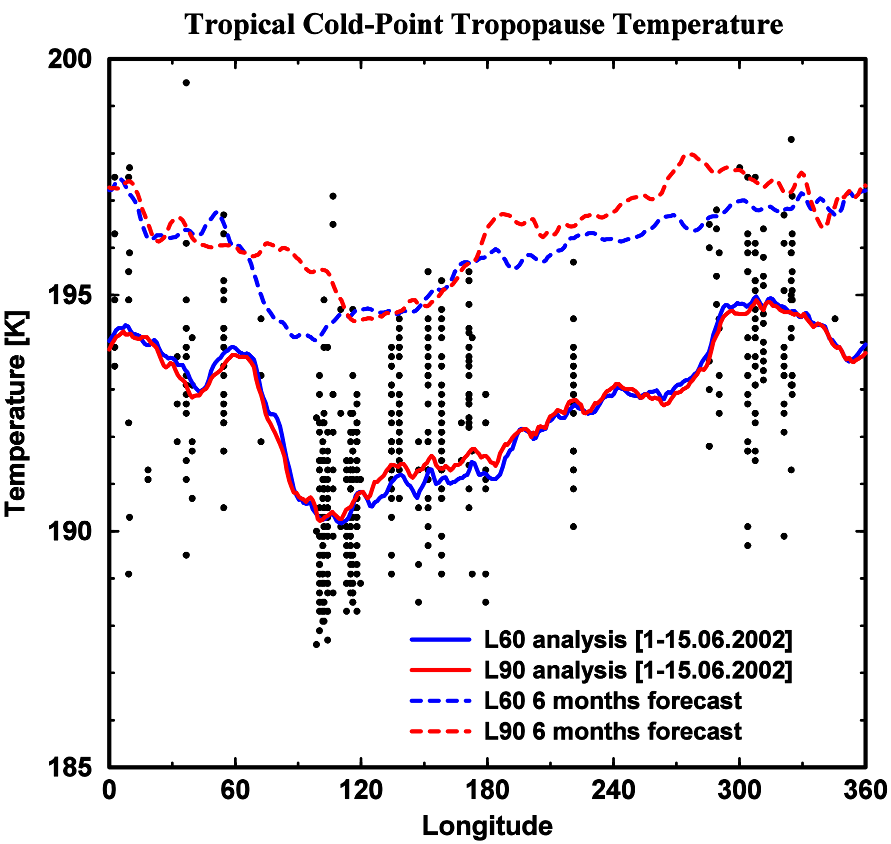
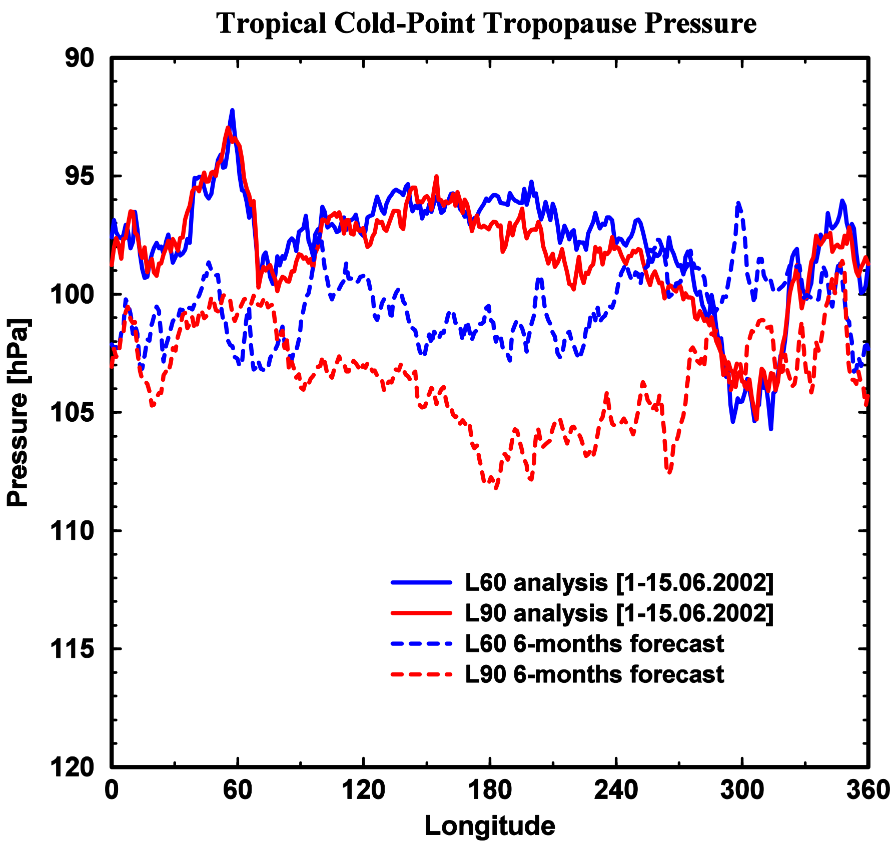
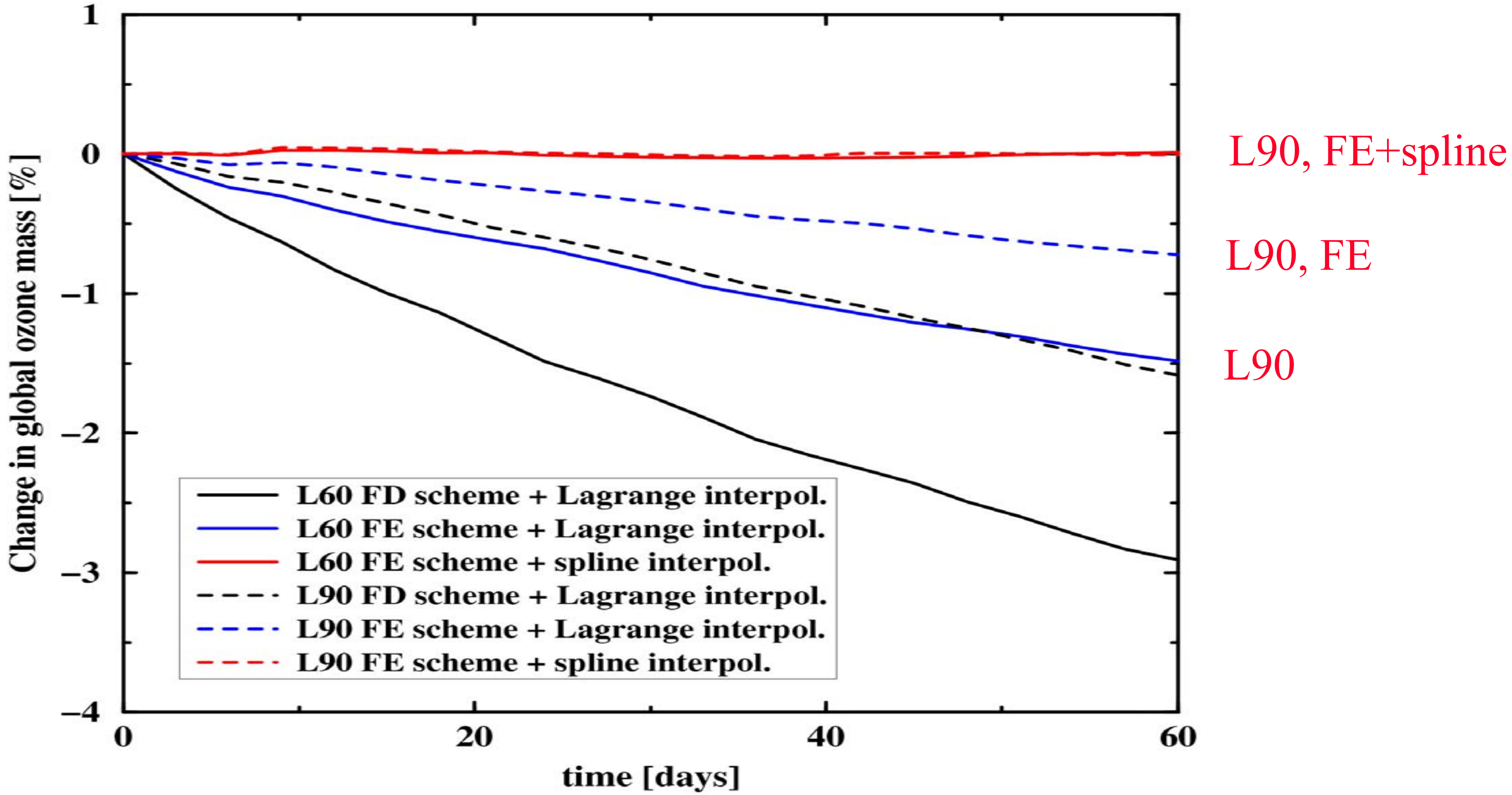
Tropical Cold-Point Tropopause in L90 / L60





Averaged over the deep tropics [10S to 10N].
Analyses and radiosondes averaged in time from 20020601 to 20020615
Forecasts averaged over whole month of June.

L90: Impact on ozone conservation



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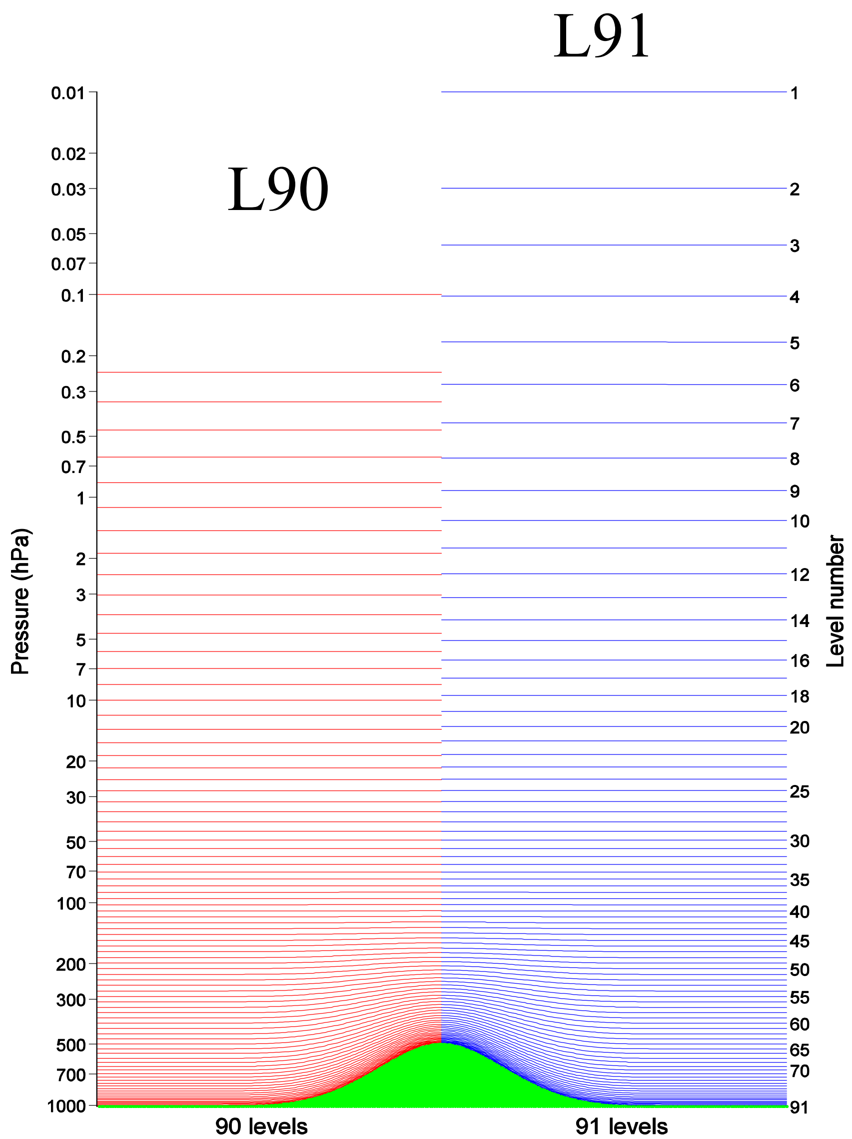
Model top raised from 0.1hPa to 0.01hPa



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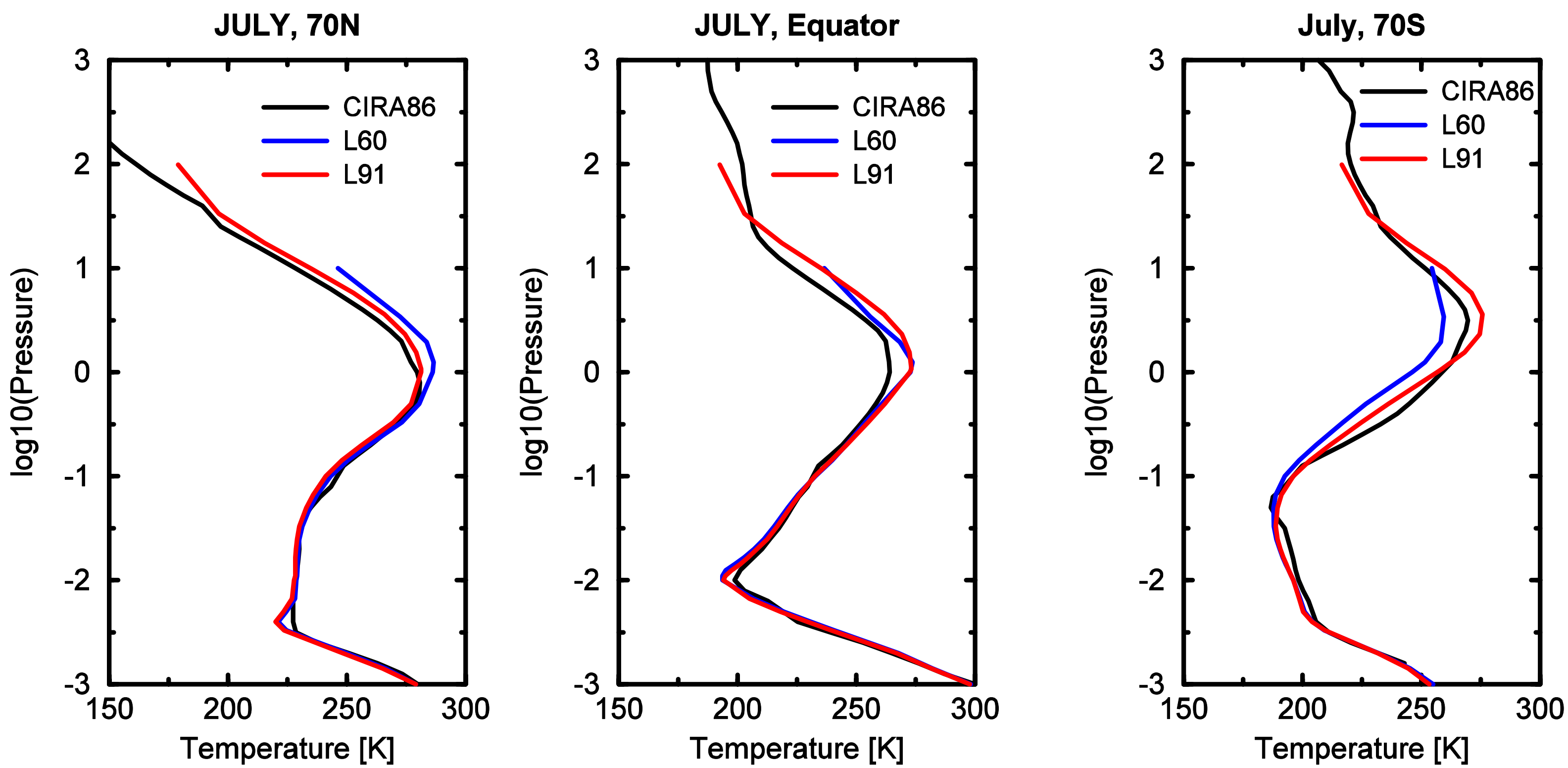
L91: Comparison with CIRA86 Climatology for July



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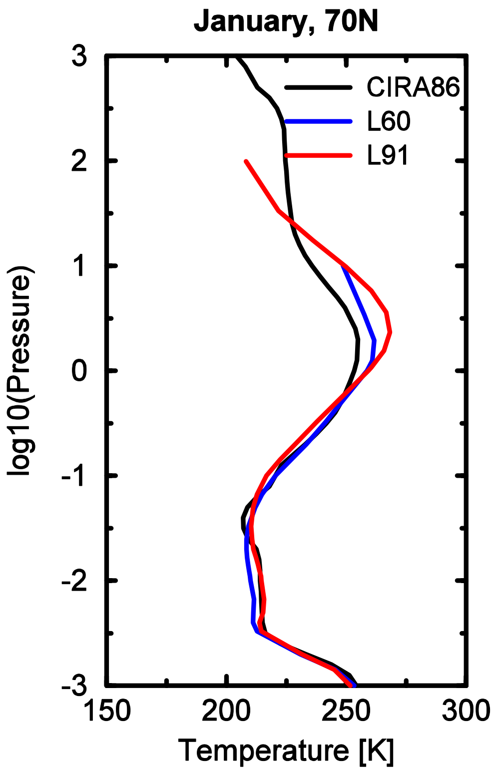
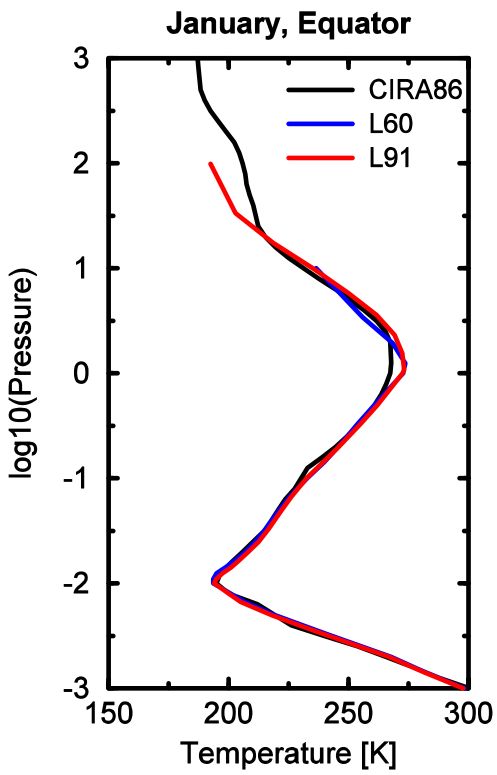
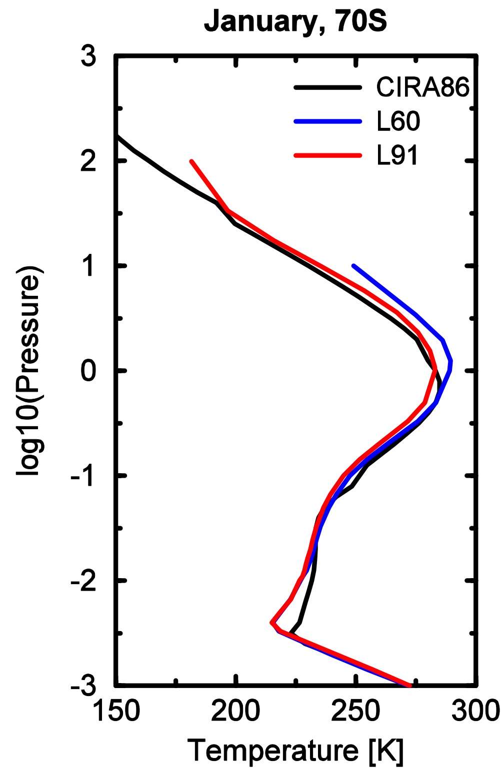
L91: Comparison with CIRA86 Climatology for January







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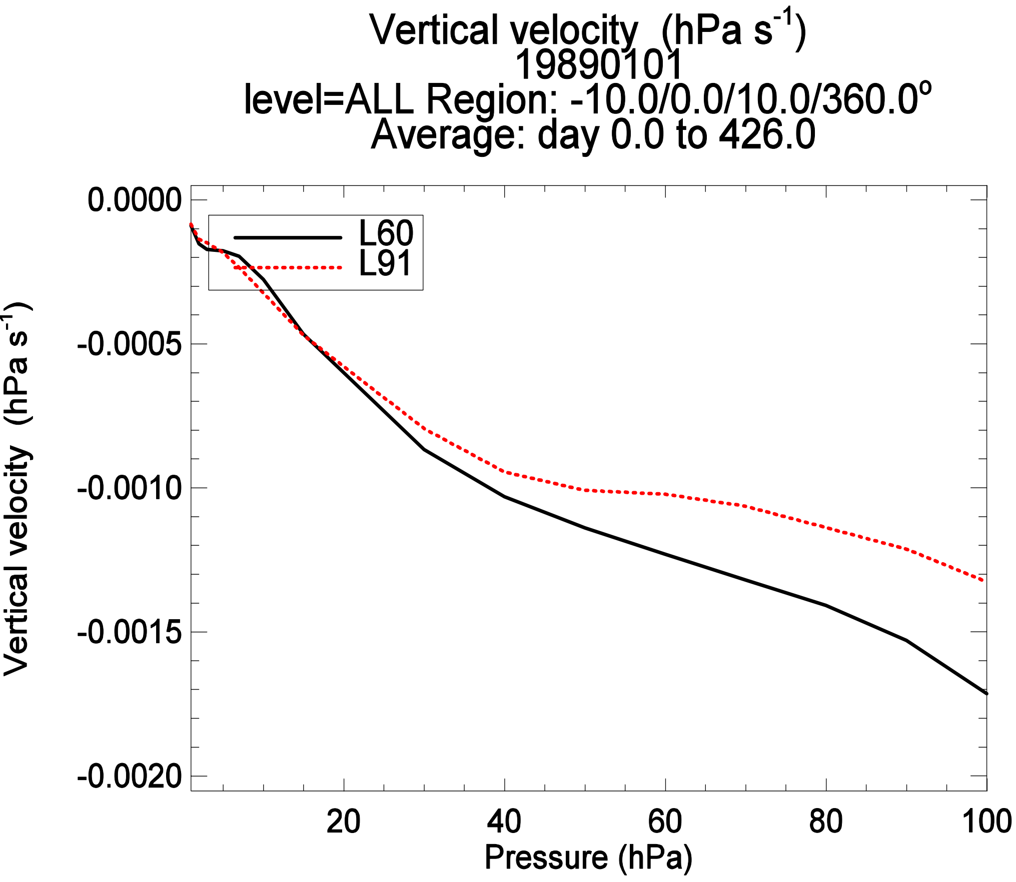
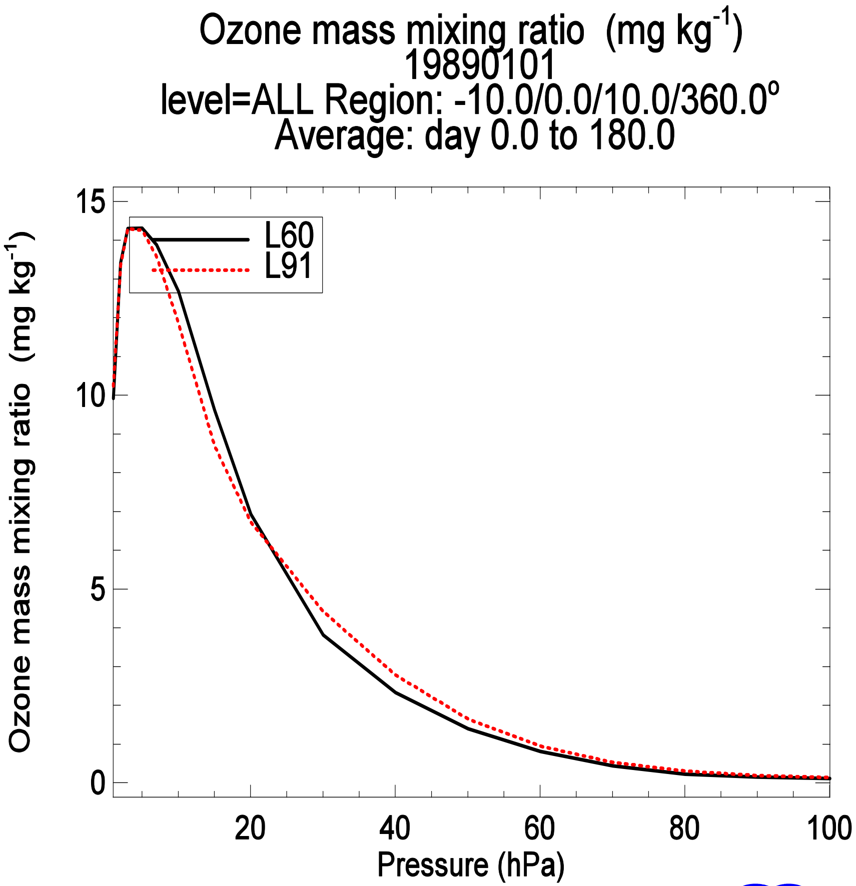
L91: Reduction in vertical velocity in the tropics



⇒ improved ozone in lower stratosphere



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Future work

- Based on the L90 or L91 model version, try to
  - improve vertical transport in the stratosphere
    - ➔ benefit for ozone assimilation and interactive ozone with radiation
  - reduce large model errors near the stratopause
    - ➔ less problems with assimilation of satellite data
- Continue work on the use of vertical spline interpolation in the semi-Lagrangian advection.
- Improve upper boundary condition (Nils Wedi).