TECHNIQUES FOR LIMITED AREA MODELLING

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1. INTRODUCTION

Many of the techniques and problems associated with limited area numerical weather prediction models are common to those of other atmospheric modelling efforts. In this study attention is confined to those aspects that are distinctive to limited area prediction models.

These latter models occupy a niche in the forecaster's repertoire between the global/ hemispheric models and the extrapolation procedures based upon an accurate local knowledge of the current state of the atmosphere. The global domain models are designed to give relatively general forecasts for periods from about 24 hrs onwards and the extrapolation techniques attempt to give detailed site-specific forecasts for 1 - 6 hrs ahead. Thus limited area models should at least seek to provide information for the intermediate forecasting time range. However their judicious use outside of this time range is also possible.

It follows, from the forecast time range, that synoptic and subsynoptic phenomena with space and time scales of 20 - 2000 km and 2 hrs - 2 days respectively (the meso-α and meso-β scales as defined by Orlanski, 1975) are features that should be represented in some detail in these models. Now meso-α and -β phenomena (e.g. fronts, jet streaks, orographic effects, surface convergence zones) possess some salient physical features: - an intimate link with the larger scale circulation, strong ageostrophic but quasi-balanced flow components, and (embedded within them) finer scale weather producing features.

The adequate representation of these distinctively mesoscale properties mark out the desired requirements of limited area models viz. fine mesh spatial resolution with corresponding (if possible) initial data sets, and large scale forcing via time dependent lateral boundary conditions. It is numerical features associated with the modelling of these aspects that are considered in the succeeding sections.

In sections 2 and 3 a critical examination is undertaken of the available techniques for the treatment of the model lateral and upper boundaries. In principle the boundary treatments should ensure the effective representation of the forcing by the larger scale circulation on the flow in the model domain, and that the model generated local flow is not adversely affected by the presence of the boundaries.

Issues related to the initialization and initial data sets for sub-synoptic scale flow are discussed in section 4. Initialization techniques for use with global models are now well developed and successful. For limited area models the initialization requirements are of central importance. However, the nature and scale of the flow, the presence of the lateral boundaries and the diverse form of the available data place different constraints upon the problem.

The material is developed from a theoretical standpoint to emphasize the underlying concepts and problems, rather than giving a presentation of some of the wide range of modelling results. In the final section brief comments are made on the possible future role of limited area models.

2. TREATMENT OF THE LATERAL BOUNDARIES

Lateral boundary formulations fall into two categories, the so-called "one-way" and "two-way" interaction methods (Phillips and Shukla, 1973). In the "one-way method" a history tape of the flow variables in a boundary region of the fine mesh domain is obtained from a prior coarser mesh, larger domain model integration. Then in the integration of the limited area model it feeds parasitically on the coarser mesh data (i.e. feedback into the coarser mesh model is excluded). For the "two-way method" the finer mesh model is embedded within the larger model area with a marginal overlap of the two domains. The models are integrated as one unit with a full coupling in the interface region.

Thus, for prediction purposes the limited area model is envisaged to be dynamically driven by, or linked to, a larger domain via the lateral boundary conditions. Now the data derived from an integration with a coarser grid over a larger domain will contain features that differ from the fields derived from the finer-mesh limited area model. These features will be associated with both numerical effects (e.g. phase speed differences, and flow field structure near the limit of the resolution of the coarser grid) and physical effects (e.g. the hopefully better defined initial data for the limited domain, the development of sub-synoptic features during the model integration).

Acknowledging these differences the lateral boundary treatment should serve a twofold purpose. It should be capable of transmitting smoothly into and out of the limited domain the large scale flow resolved by, and implicit in, the external specified boundary data. It should also adequately represent the outgoing inertia-gravity waves and fine-mesh-scale meteorological flow that might be inherent in the initial data of the limited domain or generated in situ during the time integration.

2.1. <u>One Way Interaction Method</u>

a) Background Considerations

For the one-way method the above defined role of the lateral boundaries expressed tersely requires the limited-area initial-boundary value problem to be well-posed. In essence, the flow field in the limited domain should be unique and depend continuously upon the initial and boundary data. This places constraints upon the "smoothness" of the interior flow field (e.g. Serrin, 1957). Also only boundary data corresponding to the transfer of information into the domain should be specified at the boundary (e.g. Charney, 1962).

The hydraulic jump phenomenon of the non-linear shallow-water-system (Stoker, 1957; Williams and Hori, 1970) demonstrate that this meteorological prototype system is not necessarily well-posed for all initial conditions even in the absence of lateral boundaries. For limited area models it has been shown that for the barotropic vorticity equation an infinite vorticity gradient can be advected in from the boundary if the vorticity at a boundary point of tangetial flow is not compatible with both the neighbouring inflow and outflow values (Bennet and Kloeden, 1976). Also, from consideration of a linear system consisting of hydrostatic perturbations of an uniform flow field in a bounded atmosphere, it has been argued that a purely local formulation of boundary conditions, i.e. fields not decomposed into their separate eigen-structures, will not constitute a well posed problem (Oliger and Sundstrom, 1978). For a non-rotating system, this last result follows from noting that the phase and group velocities of the two buoyancy waves associatcd with each vertical eigenfunction solution (i.e. the natural modes) is given by

$$\{ U \pm (gH)^{1/2} \} \qquad (1)$$

where U is the mean flow speed, and H, the separation constant (or equivalent depth), assumes different values for each eigensolution (see e.g. Davies, 1983). It follows that the direction of energy transport (and hence the lateral boundary condition) depends on whether $|U| \gtrless (gH)^{1/2}$. Thus the boundary specificaiton is eigenfunction dependent. It is worth-while considering the generality of this last result. The addition of rotation introduces the meteorological mode, which advects with the flow, and the phase and group velocities (u_p, u_G) of the inertia-gravity waves are given by

$$u_p = U \pm (gH)^{1/2} (1 + \epsilon^2/k^2)^{1/2}$$

$$u_g = U \pm (gH)^{1/2} / (1 + \epsilon^2/k^2)^{1/2}$$

where $\epsilon^2 = f^2/(gH)$, and k is the horizontal wave number. Thus the previous conclusions ar substantially unmodified for $\epsilon^2/k^2 << 1$.

On the other hand, the inclusion of vertical shear, without rotation, alleviates the above problem. The natural modes of the system are again split into two types. One half posess horizontal phase (and group) velocities greater than the maximum basic flow velocity, and for the other half the corresponding velocities will be less than the minimum velocity of the basic flow (see e.g. Wiin-Nielsen, 1965). This result, coupled with the expectation of the low basic wind speeds near the ground, indicates that a good a priori estimate can be made of the nature and number of variables that are needed to be specified at the lateral bounderies of this simple system.

b) Examination of particular schemes

There are a wide range of lateral boundary treatments (- of the same order as the number of limited area models!). To produce a mere catalogue of these schemes would not be particularly helpful. Instead an avowed critical examination is undertaken here of the general properties of these schemes. The treatment follows closely that of Davies (1983). Attention is given to the manner in which the schemes attempt to circumvent the overspecification problem, and also to the degree to which the solutions in the interior region are adversely effected by the boundary formulation. In particular the possibility is explored that inherent adverse computational effects can occur even if the continuous system is well-posed and its solution exhibits the required properties.

The schemes usually adopted fall in two categories - viz. boundary zone schemes that modify the flow field in a marginal zone near the boundary,and pseudo-radiation schemes that act only at the boundary itself. The formulation and behaviour of a selection of these schemes in the context of the linear advection equation

$$\frac{\partial}{\partial t} u + c \frac{\partial u}{\partial x} = 0 \tag{2}$$

will be considered. The flow system is assumed to occupy the limited domain $0 \leq x \leq L$. For c > o, the characteristic form of (2) indicates that it should be solved with u specified at x = 0, and the u field determined internally at x = L. This equation can be interpreted as an indicator of the behaviour of the meteorological flow component advected along by the mean flow $c = U$, or as representing the propagation of gravity waves with a propagation velocity $c = U \pm (gH)^{1/2}$ (e.g. Davies, 1976). We now consider the various schemes in turn.

Diffusive Damping Scheme

A marginal zone of large diffusion for the prognostic variable u is introduced in the vicinity of the lateral boundaries (e.g. Benwell et al. 1971; Burridge, 1975; Mesinger, 1977). Thus Eq. (2) is replaced by

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(\nu\frac{\partial u}{\partial x}\right) \tag{3}$$

where $\nu = \nu(x)$ assumes appreciable values only within marginal zones of width $\ell\,(\ll L)$ located at $x = 0_+, L_-$. An additional boundary condition on u at x = L can now be legitimately prescribed for this new system, because the viscous term has increased the spatial order of the differential equation.

Sharp shear zones and thermal gradients might be induced by the diffusion zone. Such a zone could be the seat of physical or computational instability. Here we consider some more basic effects.

Consider the effect of forcing a periodic disturbance of frequency ω at one end of a diffusive zone of constant ν . The solutions in such a zone take the form of two waves, propagating in opposite directions along the x-axis, and decreasing in amplitude in their direction of travel. To be effective, the scheme should allow

i) the transmission of an incoming wave at x = 0 without appreciable change of phase or amplitude, and

ii) damp the reflected wave at x = L so that it does not have appreciable amplitude on re-entering the inner domain at x = L - ℓ.

It can be shown that the criteria are met if

$$2\pi\,(\ell/\mathcal{L}) \ll \sqrt{2}\; R_e^{1/2} \tag{4}$$

and $$R_e \gg 1$$

where $\mathcal{L} = 2\pi(c/\omega)$ is the forced wavelength of wave solutions of (2) and $R_e = \frac{1}{2}(c\ell/\nu)$ is a boundary zone Reynolds number.

Thus provided R_e is sufficiently large the above criterial can be met. (This is not true if in the shallow-water equivalent of system (2) only one prognostic variable, say the velocity, is subject to boundary diffusion. Such schemes are inappropriate (Israeli and Orszag, 1981; Davies, 1983)). In practise the finite-difference representation of (3) will introduce constraints upon the width of the boundary zone ℓ, and hence on R_e.

To demonstrate the possible numerical effect we consider the same problem (i.e. forcing of a boundary zone) but now for the leapfrog cum Dufort-Frankel finite-difference representation of (3) i.e.

$$u_j^{n+1} = u_j^{n-1} - \alpha\,(u_{j+1}^{n} - u_{j-1}^{n}) - \mu\,(u_{j+1}^{n} + u_{j-1}^{n} - u_j^{n+1} - u_j^{n-1}) \tag{5}$$

where $\alpha = c(\Delta t)/(\Delta x)$ and $\mu = 2\nu(\Delta t)/(\Delta x)^2$ (6)

Proceeding as before we find that to satisfy the desirable condition at inflow requires $\nu^{*} = (\mu/\alpha) < 1$ to avoid spurious spatial growth and $2\pi\,(\ell/\mathcal{L}) \le s(1-0.9^{1/s})^{1/2}$ for a boundary zone $\ell = s(\Delta x)$ to produce less than 5 % damping of the incoming wave. This finite-difference analogue of inequality (4) is far more stringent. With s = 2, 3, 4, 5 the rhs of this last inequality takes the values 0.44, 0.55, 0.64, 0.7.

Thus in relation to criterion (i) we conclude from these inequalities that the diffusive boundary scheme is effective only if the wavelength of the basic system $(\mathcal{L})$ is considerbly longer than 6ℓ, where ℓ is the width of the diffusion zone. If this inequality is not satisfied, then the incoming flow field will be degraded by the zone and can suffer a significant amplitude reduction and minor phase modification. This short-coming will curtail the time-span for which the regional NWP model output is useful. In particular the transmission of comparatively smaller scale synoptic disturbances into the limited-area forecast domain could be adversely affected.

To consider the effects of outflow we introduce a system illustrated in Fig. 1 which will also be used later.

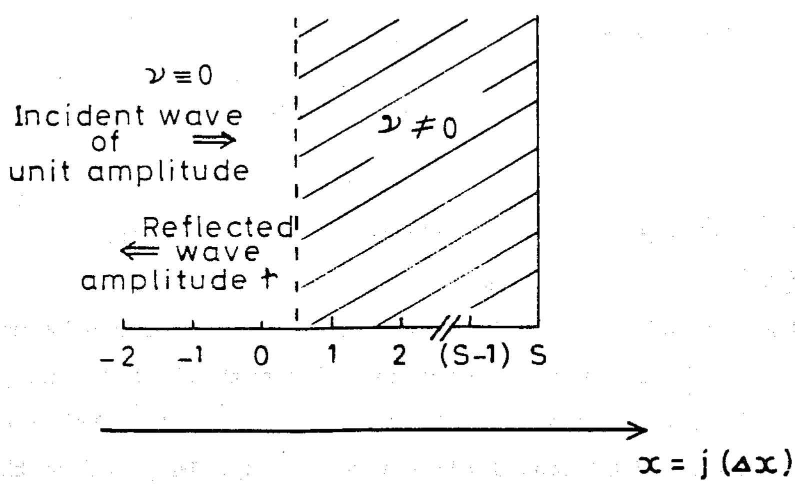


Fig. 1. Schematic depiction of the numerical flow system under consideration. A boundary zone is located in domain j = 1, s. In domain j ≤ 0 an incident wave of unit amplitude impinges upon boundary zone from $-\infty$, and a reflected wave also exists.

An incident wave of given frequency, ω, impinges from $-\infty$, upon a boundary zone $j = [1/2, s]$. The amplitude of the reflected wave in the viscous free region $j \leq 0$ will be a measure of the effectiveness of the zone. The results for various boundary widths is shown in Fig. 2.

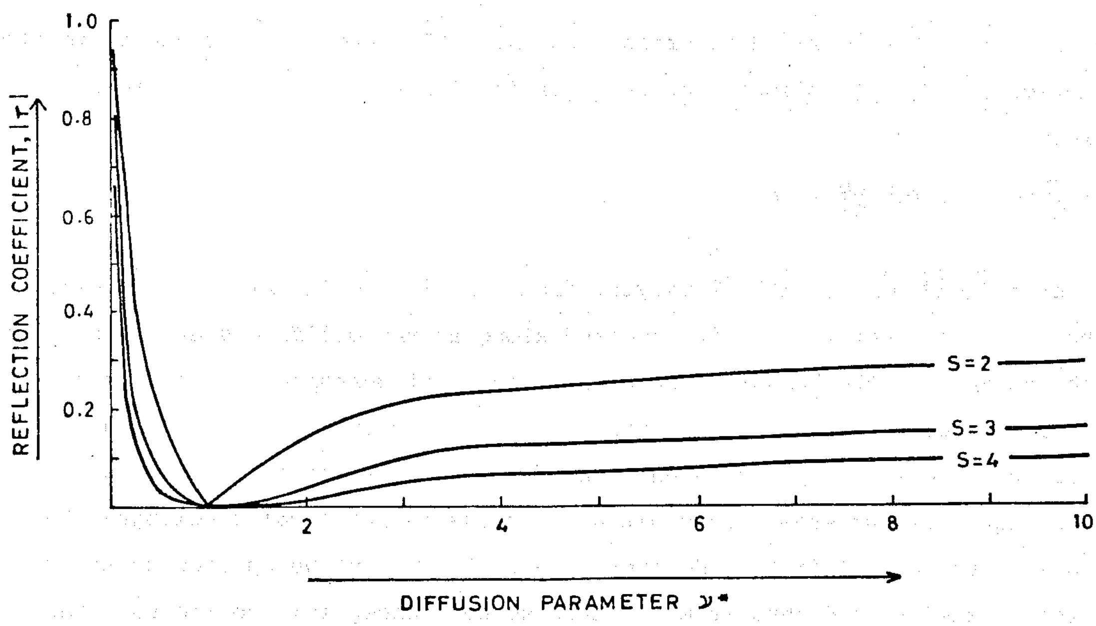


Fig. 2. A plot of the reflection (r) as a function of $\nu^* = 2\nu/c\Delta x$ in the limit of $\omega^* = \omega\Delta x/c \ll 1$ for diffusion zones of different widths with boundaries located at s = 2, 3, 4.

Appreciable reflection occurs for both small and large ν^* values. Small ν^* values induce insufficient damping of the wave and reflection occurs predominantly off the boundary $j = s\Delta x$. At large ν^* values the reflective properties of the numerical scheme gives rise to reflection at the viscous-inviscid interface.

These results indicate the inappropriateness of the difference scheme (5) as a diffusive layer. Other difference representations must be judged on their own merit, but this example serves to illustrate the potential pitfalls.

Tendency Modification Scheme

In this scheme the tendencies of the model prediction variables are modified in the marginal zone. The tendencies are assigned a weighted average of the externally specified fields and the internally determined fields such that the weighting associated with the external field varies from one at the boundary to zero at the inner extremity of the marginal zone (Kessel and Winninghoff, 1972; Perkey and Kreitzberg, 1976; Fritsch and Chappell, 1980; Maddox et al. 1981). In addition to the tendency modification the variables in the marginal zone are also subjected to a scale-selective spatial filtering procedure.

With this scheme the system represented by (2) is replaced by the equation

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = -\gamma \frac{\partial}{\partial t}(u-\hat{u}) \tag{7}$$

Here the $\hat{u} = \hat{u}(x,t)$ field is prescribed externally and, if consistent, is itself a solution of (2). It follows that the equation for the "error", $u' = u - \hat{u}$ takes the form

$$\frac{\partial}{\partial t}u' + c^{*}(x)\frac{\partial u'}{\partial x} = 0 \tag{8}$$

where $c^* = c/(1+\gamma)$, with γ varying from zero in the interior to infinity at the boundary. The error field is advected along at the modified speed c* and this reduces to zero at the boundary. Thus the problem of overspecification is again nominally overcome, but the"error energy", $(u')^2$, accumulates in the boundary zone. An incident wave of a given frequency encountering a decrease in c* across the boundary zone will undergo a concomitant decrease in its local wavelength. Hence the use of the spatial filter. However, the filter has to be applied to the u field (not the u' field), and thus it makes this scheme susceptible to the same shortcoming as that outlined earlier for the previous boundary diffusion-zone scheme.

There is also a numerical shortcoming for this scheme. For a leapfrog representation of (2) the change in the advected speed, c*, changes the "refractive index" properties of the computational system. This effect can trigger a reflected wave at the interface of different c* values. For a periodic wave that meets such a phase speed interface between domains L_1 and L_2, the resulting reflection coefficient can be inferred from Fig. 3. First, from diagram (a) estimate the change in wavelength between L_1 and L_2, given the period of the incident wave and the change in α due to the c* change. Then with this information the reflection coefficient can be inferred from diagram (b). Related effects can occur at inflow. Thus the tendency modification scheme suffers partially from the defects associated with the use of a spatial diffusion operator in the boundary zone, and from numerical transmission defects associated with c* changed in the boundary zone.

Flow Relaxation Scheme

In this scheme the prognostic variables are subjected to a forcing in the marginal zone that constrains them to relax towards the externally specified field on a time scale that again varies with distance from the lateral boundary (Davies, 1976; Kallberg and Gibson, 1977 a,b; Lepas et al. 1977; Gauntlett et al. 1978; Ninomiya and Tatsumi, 1980; Leslie et al. 1981).

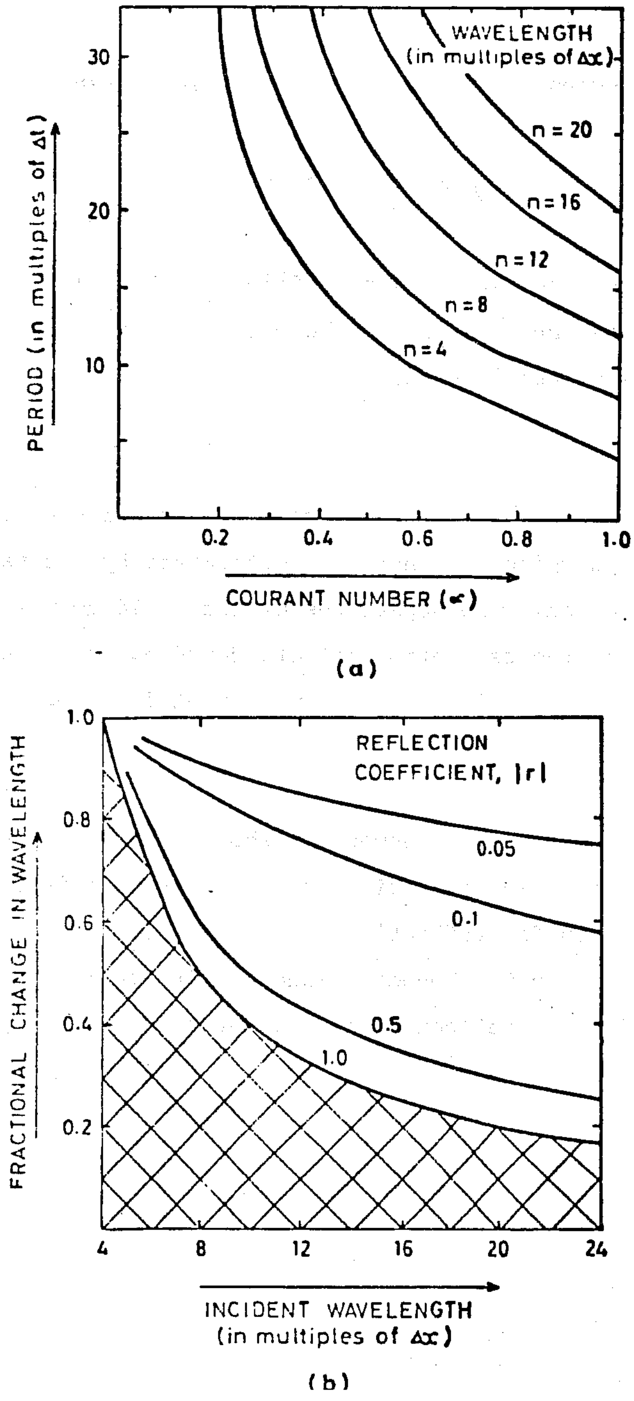


Fig. 3 The wavelength (in multiples of Δx) of the wave in L_1 or L_2 is displayed in (a) as a function of the Courant number (α) and the period of the wave. In (b) the amplitude of the reflected wave (r) is shown as a function of the incident wavelength in domain L_1 and the fractional change in wavelength (η_2/η_1) on transmission to L_2. The cross-hatched area is the region of parameter space where there is total reflection of the incident wave.

For the linear advection case (2) the modified equation for this scheme is

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = -K(u-\hat{u}) \tag{9a}$$

However, Tatsumi (1980), following a suggestion of Hovermale, extended the scheme to include a diffusion relaxation term, i.e.

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = -K(u-\hat{u}) + \frac{\partial}{\partial x}\left[\nu\frac{\partial}{\partial x}(u-\hat{u})\right] \tag{9b}$$

Here we consider (9a). Now it is the relaxation coefficient, $K = K(x)$ that is non-zero only in the boundary zones, and $\hat{u}$ is again the externally specified field. In this case the error equation takes the form

$$\frac{\partial u'}{\partial t} + c\frac{\partial u'}{\partial x} = -Ku'$$

Thus as the error field, u', is advected into the boundary zone its amplitude is reduced due to the relaxation damping at a net rate determined by the values of c and K. Note also that at inflow only the departures of the field away from the specified values are subject to the relaxation effect. Hence again in this scheme the effect of overspecification is mitigated but now without inducing a deleterious effect in the inflow zone.

However, this scheme is also not immune to numerical shortcomings. In this case also the boundary zone can induce significant spurious reflection. This is illustrated in Fig. 4 which shows the results for reflection from the system introduced in Fig.1, but now with K (not ν) constant in the boundary zone. Again small K induces insufficient damping and large K induces reflection at the interior-boundary zone interface.

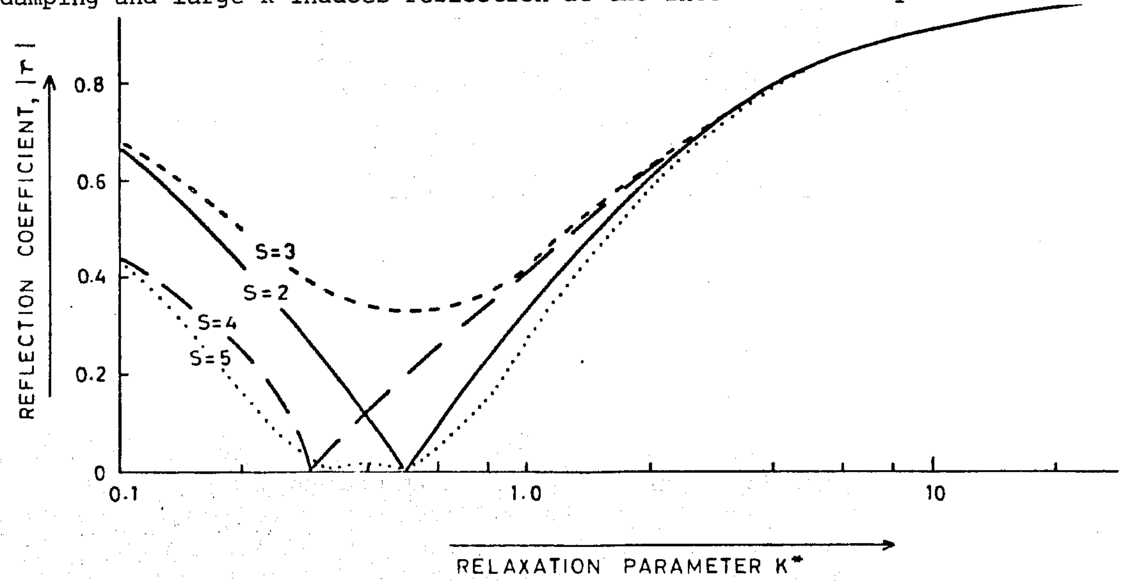


Fig. 4 Reflection (r) shown (for $w^* \ll 1$) as a function of $K^* = K(\Delta x)/c$ for relaxation zones of constant K* values but of different widths. Plots shown for boundaries located at s = 2,3,4,5 (i.e. K*non-zero, resp. at 1,2,3,4 points in the interior).

Clearly a strategy is required to allow K to vary across the zone in such a way as to avoid as far as possible both these unacceptable effects. One such strategy (Davies, 1983) is based on the fact that a spatial variation of K allows up to (s - 1) points of zero reflection along the parameter axis of $K^* = K(\Delta x)/c$. Thus a scheme can be devised to minimize the reflection within a definable band of interest. The results shown in Fig. 5 are for a K* band with $K^*_{max}/K^*_{min} \approx 10^2$. (A lower bound for K* can be established from the group velocity of the fastest wave in the model. An upper bound exist in practice since very slow moving waves will not penetrate sufficiently into the boundary zone during the period of integration to produce appreciable reflection.)

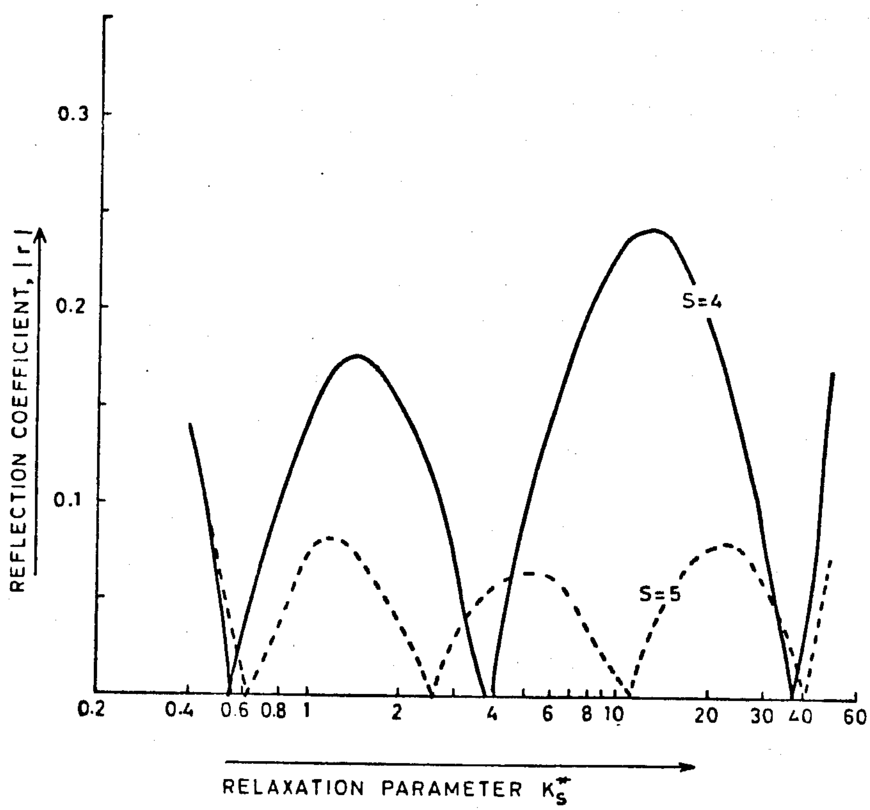


Fig. 5 A plot of the reflection (r) for $\omega^* \ll 1$ as a function of K* for relaxation zones with boundaries at s = 4 and 5, and with 'tuned' specified spatial variation for $K^*_{j=1,s-1}$

For this rather stringent system the maximum reflection in the specified range $0.47 < K^* < 47$ is less than 8%, and the reflection coefficient is less than 0.05 over most of that band. The adoption of a semi-implicit scheme would have the beneficial effect of decreasing the width of the K* band of interests and hence increase the effectiveness of the zone.

Pseudo-Radiation Schemes

These schemes differ from those already considered in that there is no direct modification of the prognostic variables in the marginal zones, but only a direct specification or calculation of the variables at the lateral boundary itself. For the system (2) the characteristic form shows that the dynamical quantity should be specified only at boundaries of inflow i.e. where c is directed into the domain. In the context of the barotropic vorticity equation this demands that the vorticity to be specified at inflow and determined internally at outflow. Shapiro and O'Brien (1970) adopted this strategy using an upstream advection scheme at the boundary. This method was adopted for a P.E. model by Williamson and Browning (1974) with the modification that all prognostic variables are specified at inflow and advected with the flow field at outflow. This procedure might be appropriate if the flow field of the P.E. model is well-balanced without appreciable inertia-gravity wave effects. A similar procedure allowing for some relaxation toward the external field at the boundary itself, has been used recently by Kurihara and Bender (1983).

An alternative approach was originally proposed by Orlanski (1976), and there have been many subsequent variants,e.g. Miyakoda and Rosati (1977), Klemp and Lilly (1978) Klemp and Wilhelmson (1978), Clark(1979), Camerlengo and O'Brien (1980), Miller and Thorpe (1981), Ross and Orlanski (1980). This approach is based on the (generally unverifiable) assumption that all prognostic variables of the set of primitive equations satisfy individually a relationship similar to (2) at the boundary. If at a given instant a boundary pseudo-advection velocity, say $c^*_{x=L}$ is directed into the domain then the associated prognostic boundary variable is specified externally, whereas the same variable is evaluated internally using a equation of the form of (2) if the flow velocity $c^*_{x=L}$ is directed out of the limited domain. The required pseudo-advection velocity is either specified externally by invoking some à priori knowledge of the flow system, or evaluated by sampling the system in the vicinity of the boundary. In the latter case some finite-difference approximation of the relationship

$$c^*_{x=L} = - \left(\frac{\partial u}{\partial t} \Big/ \frac{\partial u}{\partial x} \right) \Bigg|_{x = L-(\Delta x)} \qquad (11)$$

A detailed study of the reflection and transmission properties of this approach for a buoyancy wave system has been given by Klemp and Lilly (1978). They show that overspecification at the boundary,specification of the correct number of variables but with inaccurate values,and errors in the estimate of $c^*_{x=L}$,all contribute to partial reflection of outward propagating waves. The first two sources of error arise if a vertical decomposition of the fields is not undertaken. The schemes of Pearson (1974), Klemp and Lilly (1978), Klemp and Wilhelmson (1978) seek to use a priori knowledge of the physical system to ameliorate these effects. Better estimates of $c^*_{x\simeq L}$, based on (11), can be derived with higher order finite-difference schemes. Miller and Thorpe (1981) present a hierarchy of such schemes.

A rigorous and useful method of extending the pseudo-radiation boundary scheme to more complicated system than (2) is not readily apparent. The rationale for the approach adopted by Orlanski derives from a dictum attributable to Sommerfeld. He noted that if wave-energy is radiating away from localized sources then in the far-field the specification of the phase relationship between the flow variables should ensure an outward directed flux of energy (see for instance Sommerfield, 1949, pp. 132). The rigorous application of this requirement to only slightly more complex lateral boundary flow problems (Bennett, 1976, Beland and Warn, 1975) indicates that this pristine approach demands an inordinate computer storage of boundary data. Engquist and Majda (1977) proposed an attractive method for obtaining approximate 'local' boundary conditions that circumvents this storage requirement. For a two-dimensional wave equation their method generates a sequence of higher order boundary formulation.

The pseudo-radiation scheme (11) is the lowest order accurate scheme in this sequence. In this case the reflection is proportional to the angle of departure of the wave propagation away from normal incidence.

This follows from noting that the radiation-condition applies strictly to energy outflow at normal incidence. Thus in line with Sommerfield's reasoning the validity and hence the usefulness of a 'radiation-type' boundary scheme must be in doubt if there are energy sources (e.g. diabatic heating, frictional effects) or phase perturbing effects (e.g. orography) in the vicinity of the boundary. Lilly (1981) has shown that the pseudo-radiation scheme can nevertheless handle non-propagating, temporally growing perturbations of a simple flow system. Here we indicate one kind of error than can ensue when using the pseudo-radiation scheme in an inappropriate flow situation.

Consider the following slight generalization of (2) to include a constant forcing term (-F), i.e.

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = -F \tag{12}$$

On rewriting (12) in the compact form,

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial x}\right)(u + Ft) = 0$$

we deduce that an appropriate formalism at the boundaries is to specify $(u+Ft)$

at x = 0 and to evaluate ($u+Ft$) at x = L. However, we shall now show that this is not necessarily the outcome obtained when using the pseudo-radiation boundary scheme.

The general solution of (13) is of the form,

$$(u+Ft) = f(X) \qquad \text{where } X = x-ct$$

It follows that at the boundary the value of $c^*_{x=L}$ of (11) is given by

$$c^*_{x=L} = c + (F/f_X)_{x=L}$$

In the usual method of implementing the pseudo-radiation scheme, the value of u at the x = L boundary will be either specified or evaluated depending upon whether $c \gtrless (F/f_X)$. Overspecification will result if the lower inequality prevails, and with a numerical scheme for (11) this will result in at least transitory reflection. Again numerical problems can ensue if the upper inequality prevails with $c^*_{x=L}$ such that $c^*\Delta t/\Delta x > 1$. This computational problem is clearly divorced from the true physical problem and hence is another potential shortcoming of this approach.

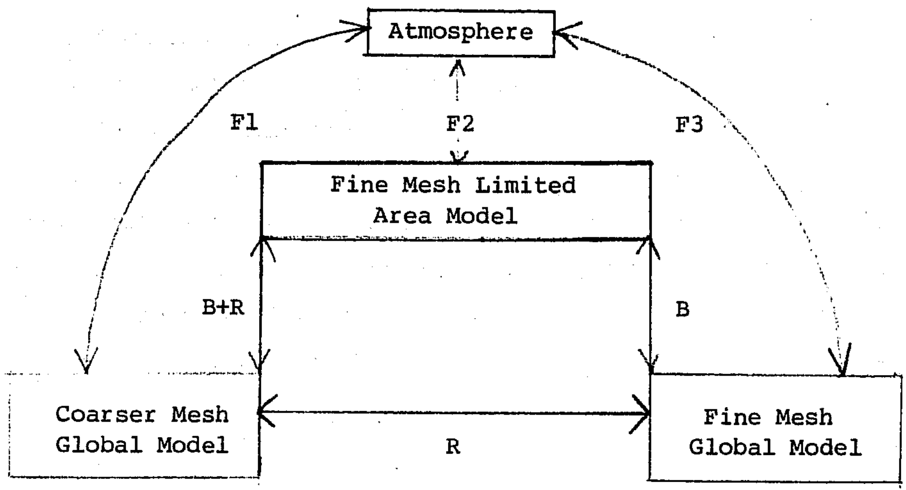
2.2. Two Way Interaction Method

The two-way interaction method is conceptually more attractive if somewhat more cumbersome to implement. It appears in principle to avoid the problem of well-posedness that arises in the one-way method. Nevertheless the unavoidable numerical differences between the models present problems. For instance phase speed differences due to grid length differences in the models can induce deleterious effect - e.g. amplitude changes can occur on transmission of a wave across the interface, or partial reflection can take place as discussed earlier. In effect although the well-posedness issue does not arise in connection with the differential system, nevertheless it occurs in conjunction with the difference equations. (An attractive approach for interpolating the data between the two meshes that seeks to preserve amplitude and phase using an amplitude restoration device is that of Shapiro (1978)).

The two-way strategy has been used to good effect in studies of hurricanes (Harrison,1973; Jones, 1977; Kurihara and Bender, 1980). In these models the grid system is movable and the storm is kept within the fine mesh. This circumstance reduces the effect of the previously mentioned limitation. This device, of a movable grid system is rendered less effective in mid-latitudes because of the spatial variability of synoptic and sub-synoptic flow.

2.3. Evaluation of Schemes

There have been comparatively few published systematic evaluations of boundary treatments in a forecasting context. There are however many examples of single synoptic case successes. A thorough study to establish a justifiable case for these models would include many of the ingredients in the following schematic:



where the interlink comparisons indicate the following,

F1, F2, F3 are the actual forecast errors

R is a model-model measure of resolution errors

B is a model-model measure of the boundary treatment

B + R is a model-model measure of the boundary resolution errors.

The recent study of Baumhefner and Perkey (1982) involved such a set-up, and they tested the tendency modification scheme (PK) and the Williamsen-Browning (WB) (1974) variant of the pseudo-radiation scheme. They report that for the particular models used by them

(a) $B \neq 0$. The errors generated are attributable to the boundary diffusion scheme and the interpolation required to a staggered fine mesh grid.

(b) The B and R model-model components were comparable.

(c) F1 and F2 can be comparable, but the ratio F1/F2 is highly case dependent.

(d) The B+R errors are associated both with the development of rapid transients and a major component that propagates into the domain at 20-30° longitude per day and is 'related to a loss of amplitude in synoptic features as they enter the domain'. These errors increase with an increase in the diffusion specification (c.f. our earlier comments).

(e) The WB and PK scheme behaviour is qualitatively similar but the WB scheme can suffer from an instability problem in the vicinity of tangential flow at the boundary.[Orlanski (private communication) has also deduced that F1 and F2 are comparable. He attributes this result directly to the boundary resolution errors].

3. THE UPPER BOUNDARY

The issue of the upper boundary has been a long term enigma for numerical modellers.

The upper atmosphere acts both as an absorber and reflector of energy propagated from lower altitudes. In principle, the formulation of atmospheric numerical models should be in accord with this fact. Thus an upper boundary condition applied at some finite height should allow for the possible transmission of energy through that level. This energy would correspond to that which in the real atmosphere would be absorbed at altitudes above the top of the model.

An upper boundary condition that requires the vertical velocity (w) or a pseudo-vertical velocity (ω or $\dot{\sigma}$) to be set to zero at some finite height, pressure, or pseudo-pressure level will effect a perfect reflection of wave energy at that level. Again, due to truncation effects, setting ω or $\dot{\sigma}$ to zero at the model's level of zero pressure will also induce reflection.

Atmospheric wave theory results indicate that both planetary scale Rossby waves and meso-scale inertia-buoyancy waves can propagate energy vertically. To the present, limited area forecast models have not usually allowed for an 'open' upper boundary. If the vertical propagation of inertia-buoyancy waves is invariably inhibited in the atmosphere by the shear of the mean flow, and/or non-linear effects, then such a boundary condition may indeed be superfluous. Simple theoretical models tend to indicate that the energy and momentum redistribution achieved by vertical propagating waves can be significant.

Specialized meso-scale models disigned to study particular phenomena (e.g. strong lee-side winds) have incorporated an upper boundary treatment. For instance a technique analogous to the diffusive boundary zone and the relaxation scheme were employed by Houghton and Jones (1968) and Klemp and Lilly (1978). The latter authors suggest that due to computational reflection it is desirable to have of the order of eight model layers in the damping zone. This computational requirement indicates that such a scheme would be unacceptable for most forecasting models.

This predicament provides the motivation for seeking a radiation type boundary condition. The strict application of such a condition in a time dependent problem requires knowledge of the time-history of the flow structure across the entire domain (Beland and Warn, 1975, Bennett,1976). It is therefore too cumbersome for implementation in most forecast models, and moreover its validity for the nonlinear atmospheric situation is in doubt

The application of the Orlanski (1976) technique is another possibility, although its usefulness in this context has been questioned by Mason and Sykes (1978). They state that this scheme's treatment of obliquely incident waves is ineffective and that it produces a false vertical momentum transport in evanescent wave situations.

Here we outline one further approach. Hydrostatic wave perturbations of an anelastic, incompressible system in uniform motion take the form

$$w' = \exp\{i(kx + \ell y - \omega t)\}\,[A \exp(-inz) + B \exp(inz)], \tag{14}$$

where ω satisfies the dispersion relation

$$(\omega - Uk)^2 = \{f^2 + N^2(k^2+\ell^2)/n^2\}, \tag{15}$$

and the two terms in (14) correspond respectively to upward and downward energy transmitting waves. The downward propagating mode will not be excited at a horizontal boundary if

$$\frac{\partial}{\partial z} w' = -in w' \tag{16}$$

To proceed further note that energy propagation is related to the correlation of w' and p'. Thus it is appropriate (Davies, 1980) to seek a boundary condition for w' as a function of the density scaled pressure perturbation, i.e.

$$w' = f(p'/\bar{\rho})$$

Now from the linearised perturbation equation

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\frac{\partial}{\partial z}(p'/\bar{\rho}) = -N^2 w' \tag{17}$$

Hence for $(p'/\bar{\rho})$ also of the form of (14) we have for a given wave mode

$$w' = \{(\omega - Uk)n/N^2\}\,(p'/\bar{\rho})$$

and using (15) this can be written as

$$w' = \{[f^2n^2 + N^2(k^2+\ell^2)]^{1/2}/N^2\}\,(p'/\bar{\rho}) \tag{18}$$

This relationship was recently exploited by Klemp and Durran (1963). In effect they determined the pressure field along the boundary by first performing a Fourier Transform of the vertical velocity, then used (18) with f = 0 and finally made a reverse transform to get the required field. They also proposed the use of this simplified form of (18) for rotational and nonhydrostatic systems, and presented the results of some test case studies that supported this suggestion. This technique is comparatively easy to incorporate in a model and the additional computational effort is modest. Bougeault (1983) adopted a similar approach.

Note in passing that (17) itself constitutes a boundary prediction equation. An alternative possibility to obtain a local condition is to recognize, in the spirit of Enquist and Majda (1977), that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(p'/\bar{p}) = -(k^2 + l^2)(p'/\bar{p})$$

for every individual (k, l) wave component. Then an approximation for (18), with f = 0, is the relation

$$w' \simeq \left[-\nabla_h^2 (p'/\bar{p}) \big/ (p'/\bar{p})\right]^{1/2} (N)^{-1} \cdot (p'/\bar{p})$$

It is to be stressed that these essentially impedance matching techniques are based on linear, hydrostatic Boussineq flow systems and there is an implicit assumption that the disturbances to the mean state are spatially periodic at the upper boundary.

4. INITIAL DATA AND INITIALIZATION

Mesoscale forecast models operate in the time range, outlined earlier, between that of global forecast suites and local extrapolation techniques. Thus to be effective their initialization procedures should ideally ensure that the model forecast is not hampered by spurious imbalances even in the first few hours of the forecast. Indeed it is only if this objective is achieved that these models can become viable tools for very short range (- 6 hrs) forecasts.

The various types of initialization schemes are outlined by Haltiner and Williams (1980), and we shall refer here only to aspects that relate specifically to mesoscale forecast-models. Static initialization procedures determine the wind field from the mass field, or vice versa. For atmospheric circulation phenomena with length scales (L) greater than the Rossby radius of deformation, $L_R = (gH)^{1/2}/f$ geostrophic adjustment studies indicate that the wind field adjusts to the mass field. Thus in this case it is appropriate to specify the mass field. For $L < L_R$ the reverse is true. On the mesoscale the speed of internal gravity waves is such that $L \lesssim L_R$. Thus in their mesoscale model Anthes and Keyser (1979) employ wind information to deduce the mass field. Again on the mesoscale the diabatic effcct can be a significant factor in the forcing terms of the ω equation. Hence Tarbell et al. (1981) incorporate this effect into their initialization scheme. It is useful to note the limitations of the geostrophic adjustment arguments. For instance the definiction of L_R itself should be modified in regions of strong baroclinicity and/or horizontal shear(Van Tuyl and Young, 1982). Again for some mesoscale convective or strongly forced/dissipated systems the concept itself may be inappropriate. The adjustment studies of Paegle (1978) and Carpenter and Lowther (1982) is suggestive in this respect.

Application of the normal mode methods for limited areas is hindered by the non-spatial periodicity of the spatial domain. Parrish (1980) and Wergen (1981) produced useful results using sophisticated models by merely assuming periodicity for the mode expansion. Briere (1982) performed a normal mode expansion of field variables, $\{\phi' = \phi - \phi_1\}$ that represent the deviation of the flow away from the fields $\{\phi_1\}$ that are equal to the total field $\{\phi\}$ on the boundary and satisfy Laplaces equation in the interior. Herzog and Meyer (1983) perform a mode expansion that yields boundary forcing terms that also require a suitable initialization.

The increased importance of diabatic and orographic effects in the mesoscale suggest that their role should be incorporated into the initialization procedure. (Briere detected evidence of unbalanced mountain related wave activity in his results.) Orography may be the location of genuine large amplitude inertia-buoyancy wave activity i.e. there is a local forcing of these modes and this may hinder the required convergence properties of the two well-established non-linear normal mode initialization procedures of Machenhauer (1977) and Baer and Tribbia (1977).

The related 'Bounded-Derivative' method (see e.g. Kasahara, 1982) has recently been applied to an open boundary shallow water system (Browning and Kreiss, 1982). Again the boundaries introduce further constraints i.e. the boundary data must also be on the so-called " slow manifold". Note also that boundary zone treatments of the lateral boundaries should also be catered for in the initialization procedure.
Another factor to be considered in relation to the initialization is the new range of observational data that forms the base for the short-range extrapolation forecasting procedures. In principle it is also available for the mesoscale forecast model (Kreitzberg, 1979). This data set can include
- satellite derived cloud images, in various spectral bands, at short time inter-

vals and with spatial resolution of the order of 10 km, or so.

- a network of conventional ground based radar providing a composite fine resolution coverage of the areal distribution of precipitation is an almost quasi-continuous mode.
- a network of automatic surface stations with perhaps a meso-β scale spatial density, and providing data at frequent time interval ($\sim$10 min.).

The data volume per hour from such a system exceeds by two orders of magnitude that for global scale forecast systems (Beran and Macdonald, 1982). The exploration of the problem of integrating the various types of data in a consistent way to provide a "state of the atmosphere" picture has hardly began.

An attractive idea is to employ a mesoscale forecast model running in a 4-dimensional data-assimilation mode as a vehicle to achieve this consistent composite picture. Complex features of this procedure can be conceived (e.g. the use of the surface rainfall measurements and radar data to supply the free-atmosphere latent heat input). However the feasibility of such a scheme remains to be examined since little mesoscale modelling work has been undertaken in this field. The 4-dimensional assimilation schemes using modified forms of repeated data insertion (Hoke and Anthes, 1976; Davies and Turner, 1977) are readily implemented. However they should be accompanied by suitable objective analysis scheme and also care must be exercised in the choice of the relaxation coefficients to optimize the rate of the dynamical adjustment.

5. FINAL REMARKS

Mesoscale limited area forecast models are usually viewed as potentially useful tools for the forecast time span of 6 - 24 hours. A range of numerical techniques now exist to treat the distinctive problems of such models. Thus definitive tests of their capabilities could be undertaken for an even broader forecasting timespan. In addition the possibility should be explored that the models, operating in a 4-dimensional data assimilation mode, could be used as vehicles to consistently blend the various new data forms. The results of Danard (1977) and Danard and Thompson (1983) using a simple one-level model in a semi-diagnostic mode to deduce surface winds in complex terrain is at least a pointer to the potential of mesoscale models as a diagnostic tool.

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