Diagnosing the Causes of Bias in Climate Models: Why is it so Hard?

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Abstract

The equations of climate are, in principle, known. Why then is it so hard to formulate a bias-free model of climate? Here, some ideas in nonlinear dynamics are explored to try to answer this question. A proposal is made to try to advance the science needed to develop a bias-free climate model. The proposal utilises powerful diagnostics from data assimilation. However, it is shown that these diagnostics will not identify all sources of model error, and a so-called 'bias of the second kind' is discussed. This latter bias may be alleviated by recently developed stochastic parametrisations.

1. Introduction

Climate prediction models play a crucial role in today's society, governing for example decisions on whether to prepare regionally for climate-related malaria epidemics, or the extent to which the global community commits to radical reductions in greenhouse gas emissions.

However, such models are not perfectly faithful representations of reality. Take any climate model, integrate it for a period of years so that it has asymptoted to its climatology, and it will be relatively easy to find specific meteorological variables which are biased against corresponding observations. Examples might include seasonal-mean upper tropospheric temperature in high latitudes, near-surface winds over the equatorial Pacific, or rainfall in the Asian monsoon region.

It is easy to identify these biases, but it is another matter to determine the cause of these biases. In Section 2, we discuss a specific example where what appears to be a climate bias associated with excessive drag over the ocean, is in fact associated with insufficient drag over land. In Section 3, this class of problem is illustrated in a much simpler setting, using the *Lorenz (1963)* system. The results from this study point us to a rather generic perspective on the notion of model bias, based on two rather generic features of dynamical systems: the fluctuation-dissipation theorem and the non-self adjointness of linearised dynamical operators. This leads us to a novel perspective on why it has proven so hard to eliminate model bias in climate models. In Section 4, a potential solution to the problem is discussed based on the method of analysis increments. For this method to be viable in practice, the model in question should have a data assimilation capability. Currently not many climate models have such capability.

This proposal is not, however, a panacea. In section 5, a type of bias is discussed which cannot be diagnosed from analysis increments - one related to the functional representation of sub-grid processes as deterministic. This 'bias of the second kind' is very much associated with the issue of structural uncertainty in weather and climate models, and has led to the development of stochastic rather than deterministic parametrisations[1].

2. Climate models and Jeffrey's theory of the westerly flow

As mentioned in the Introduction, comprehensive weather and climate prediction models play a crucial role in society today. One would like to incorporate as much as possible of the (partial differential equation) laws of physics into these models. But high-resolution models are computationally expensive: a doubling of resolution can result in an increase in computing time of up to 2^4 (the exponent representing the dimension of space-time). What is a minimal resolution needed if we are only interested in simulating scales of, let's say, a thousand kilometres or more? Perhaps a model with grid point spacing of a few hundred kilometres might be sufficient.

Figure 1a shows the climatological boreal winter surface pressure in Northern and Southern Hemispheres (based on ECMWF analyses) whilst Fig 1b shows the equivalent fields simulated by a (Met Office) climate model (c. 1980s) run with a grid with 5˚ spacing in the longitudinal direction and 7.5˚ in the latitudinal direction.

If one was to make a very broad-brush diagnosis of this simulation, one might say that the pressure distribution was not too unrealistic in the Northern Hemisphere, but was very poor in the Southern Hemisphere: by geostrophy, one could deduce from Fig 1b that instead of 'roaring forties', the model has 'whispering forties'. Since most of the Southern Hemisphere surface is ocean, perhaps one might conclude that the cause of the model bias lies in an overestimation of the model's drag coefficient over sea.

However, instead of pursuing this idea further, suppose a further set of simulations is performed at the higher horizontal resolution with grid point spacing is half that in Fig 1b. The results are shown in Fig 1c. Without any change to the oceanic drag coefficient, the Southern Hemisphere surface pressure distribution is now simulated quite well. However, a price is paid for this improvement: the Northern Hemisphere surface pressure simulation deteriorates and the corresponding surface flow is excessively westerly.

What is going on? First, let us recall *Jeffrey*'s *(1926)* seminal contribution to the theory of the atmospheric general circulation: the surface winds in midlatitudes are maintained against friction by a poleward flux of angular momentum from lower latitudes, generated by the extratropical weather disturbances. Although at the time this idea seemed contrary to the notion that turbulent fluxes should be downgradient, development of the theory of baroclinic instability confirmed Jeffrey's proposal. In particular, the flux of angular momentum needed to maintain the midlatitude westerlies against frictional drag can be generated provided that the baroclinic eddy troughs and ridges have a tilted north-east/south west orientation. Resolving such tilted structures in a numerical model may require significantly more resolution than correspondingly structures without tilt. This was indeed found in the idealised baroclinic lifecycle experiments of *Simmons and Hoskins (1976)*.

[1] Also submitted to a special issue of Geophysical and Astrophysical Fluid Dynamics in honour of Raymond Hide's 80th birthday.

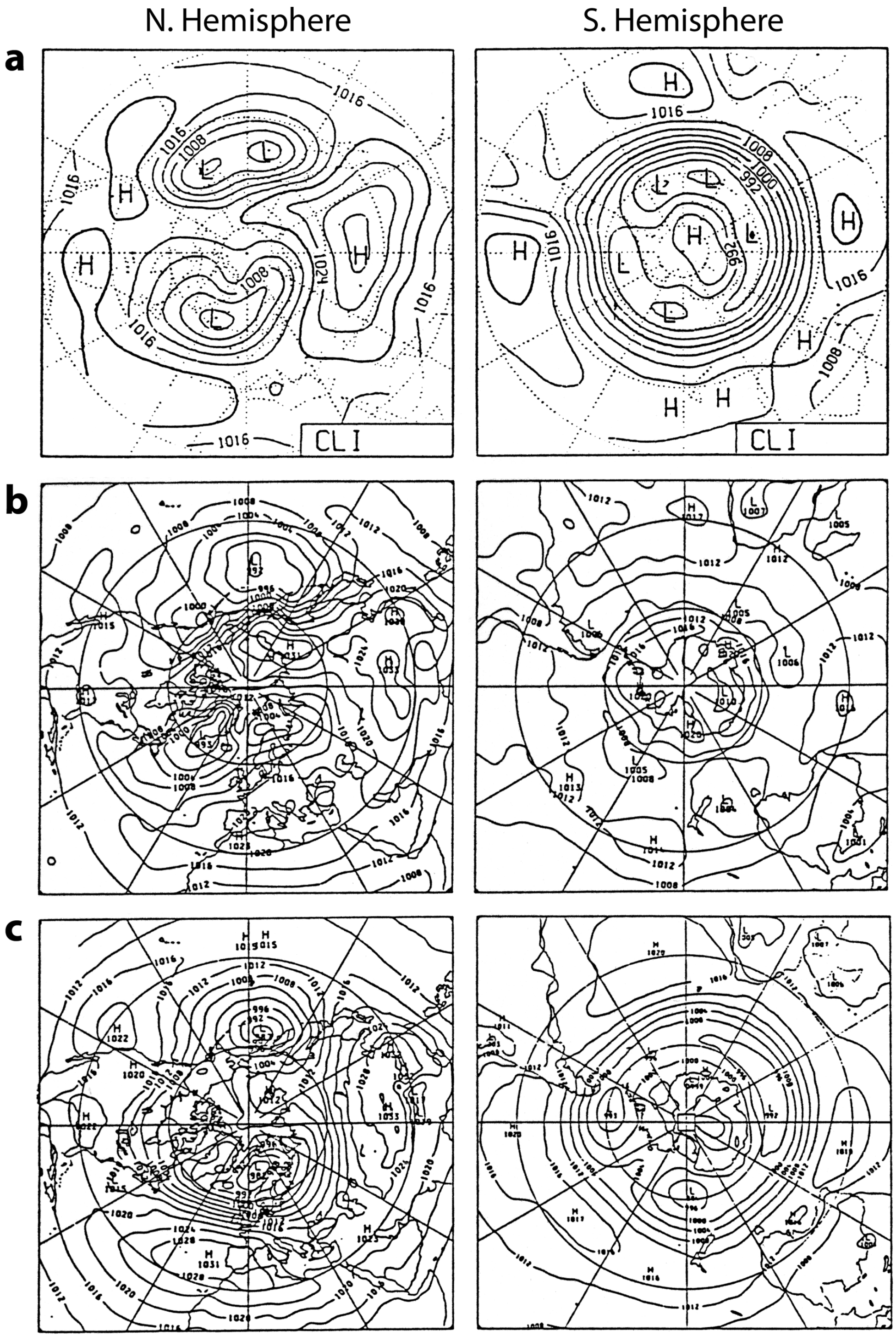

Figure 1: Boreal winter (December to February) surface pressure for Northern Hemisphere (left hand column) and Southern Hemisphere (right hand column). a) from observations, b) from a low resolution model simulation c) from a model whose resolution is double that in b).

Hence, by increasing model resolution, the ‘whispering forties’ bias has been cured. But why then, has Northern Hemisphere bias been worsened by increasing resolution? The answer is that in the Northern Hemisphere, diagnosis of the causes of bias in the low resolution model is compounded by a compensation of errors. Not only does the model have an inadequate representation of angular momentum flux associated with the baroclinic eddies, it also had an inadequate representation of frictional coupling to the land surface. In particular the versions of the model shown in Figs 1b and 1c had no parametrised representation of momentum coupling of the atmosphere to the solid earth associated with unresolved orography. Since part of this momentum coupling is in the form of orographically-forced gravity waves, a parametrisation of the vertical propagation and breaking of such waves in the lower stratosphere was developed once the biases of 2˚x3˚ (or T42) and higher-resolution models became apparent (*Palmer et al, 1986*).

Hence, what began as a suggestion that our model had too much drag over the ocean, ended by concluding that the real problem was insufficient drag over land! Along the way, it was found that inadequate representation of horizontal baroclinic wave fluxes was compensating for insufficient representation of vertical orographic gravity-wave fluxes. How could two completely different processes have such compensating effects on the climate of the model?

3. The forced Lorenz model and the fluctuation-dissipation theorem

To pursue this question further, consider a much simpler system, the ‘forced’ *Lorenz (1963)* model:

$$\begin{aligned} \dot{X} &= -\sigma X + \sigma Y + f\cos\theta \\ \dot{Y} &= -XZ + rX - Y + f\sin\theta \\ \dot{Z} &= XY - bZ \end{aligned} \tag{1}$$

where $\sigma = 10,\ r = 28,\ b = 4/3$, and a vector $\mathbf{F} = (F_{X,}F_Y, F_Z) = (f\cos\theta, f\sin\theta, 0)$ has been added to the canonical Lorenzian equations. Like the weather and climate models discussed above, the system (1) is chaotic for sufficiently small f and therefore has limited predictability.

Let us suppose that the ‘true’ system is the unforced Lorenz system (with $f = 0$), and our ‘model’ for the true system has a systematic error, represented by $f = 10$ and some $\theta = \theta_0$. Our job is to perform diagnostics of the model to determine the value of f and θ_0. Here we focus on the angular variable θ_0.

Figure 2a shows a time series of the X component of the unforced Lorenz model. Figure 2b shows a time series of the X component of the state vector of the model equations. As can be seen, the probability of finding the state in the regime with positive X is greater than what it would be from ‘observations’ of the ‘true’ system (Fig 2a). Hence, a reasonable guess would be that $\theta_0 = 0$. The time series of the X component generated with $\theta_0 = 0$ is shown in Fig 2c, and indeed it does resemble, statistically at least, the time series in Fig 2b.

However, it turns out that (in Fig 2b) our biased model of the Lorenz system has $\theta_0 = 3\pi/4$. In particular note that the actual value of F_X is in fact opposite in sign to the value associated with the guess $\theta_0 = 0$.

Figure 3 summarises the link between θ and the direction ψ associated with the time-mean response of the model to the forcing $\mathbf{F}$, i.e. the angle to the X axis of the line which joins the time-mean state of the forced model and the time-mean state of the canonical unforced Lorenz model, by definition the origin in the $X-Y$ - plane. Figure 3 shows that, for a selection of values θ, there is a tendency for the response to point along the diagonal in the $X-Y$ - plane where $\psi=\pi/4$. Generally speaking, the response does not lie exactly along this line, but to a first approximation it does. What is special about this line? It can be shown (*Selten, 1995*) that this line corresponds to the dominant Empirical Orthogonal Function (EOF) of the standard unforced Lorenz model, that is, it corresponds to the leading eigenvector of the lag-zero covariance matrix of the (three dimensional) state vector of the unforced system.

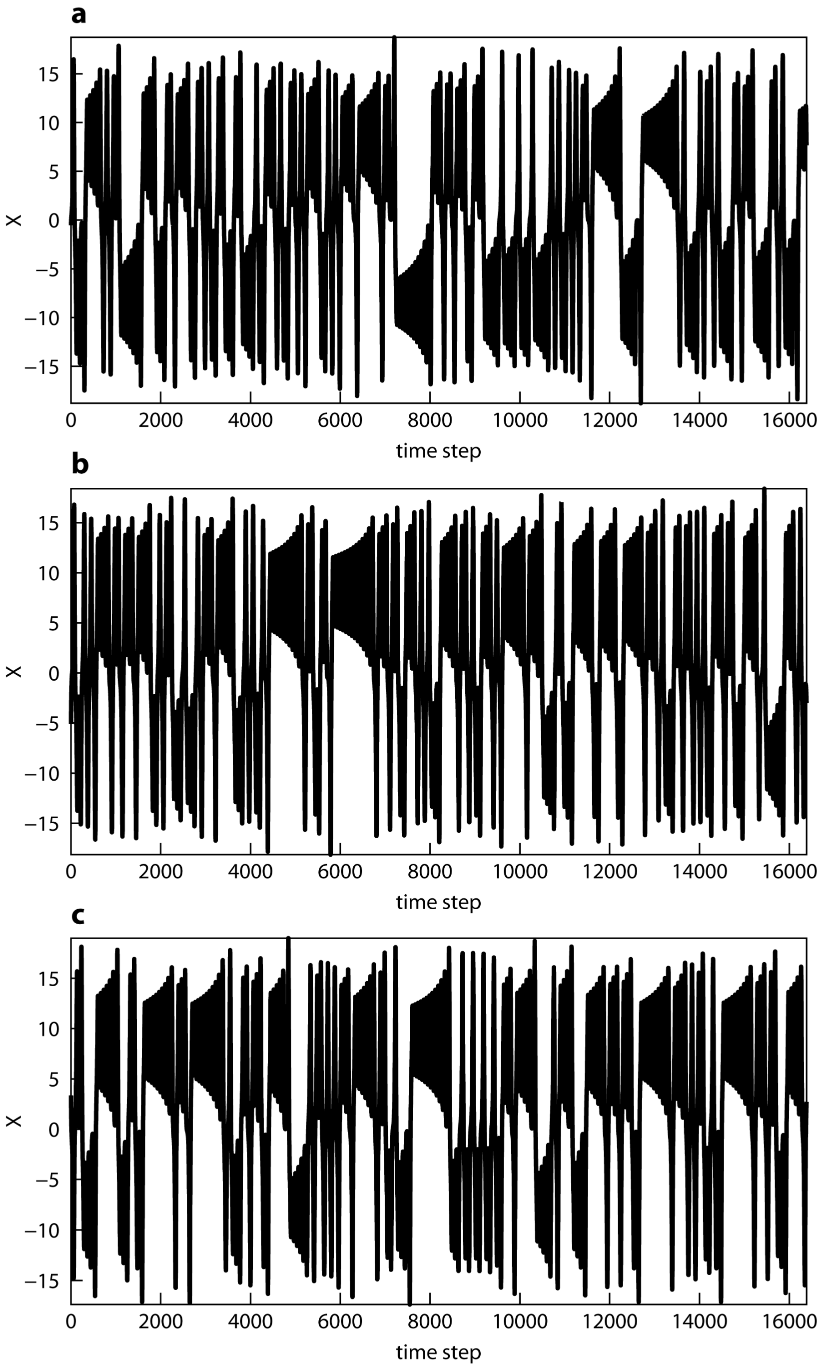

Figure 2: Time series of the X component a) of the unforced Lorenz system, b), c) of the forced Lorenz system (equation (1)) for $\theta=3\pi/4$ and $\theta=0$ respectively.

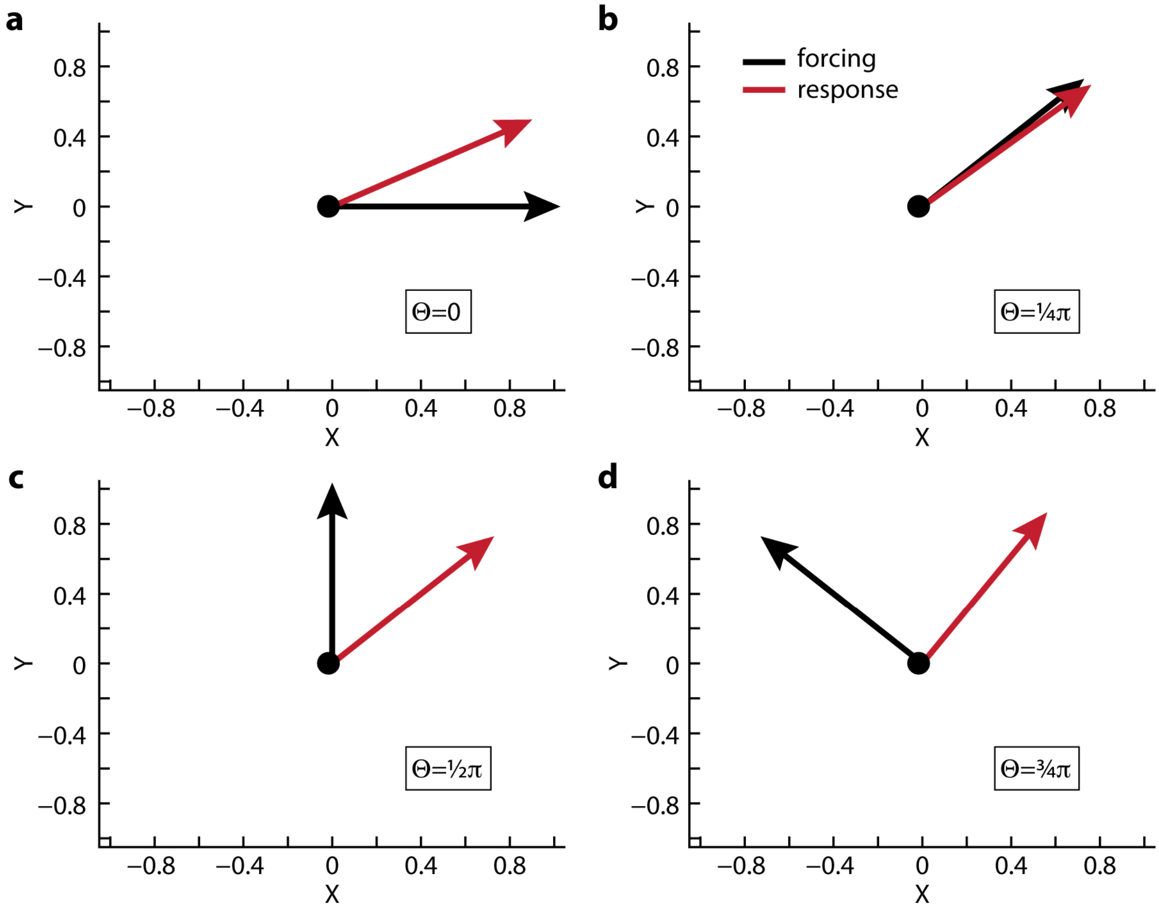

Figure 3: Time mean response to forced Lorenz system (red arrow) in relation to direction of forcing (black arrow) in X-Y plane. a) $\theta = 0$*, b)* $\theta = \pi/4$*,c)* $\theta = \pi/2$ *,d)* $\theta = 3\pi/4$.

This opens a possible link to the example in the previous section. It is well known that the so-called annular modes correspond to dominant EOFs for the atmosphere. The Northern Annular Mode, or Arctic Oscillation, (*Thompson and Wallace, 1998*) corresponds largely to fluctuations in the zonal wind in the Northern Hemisphere, with the Southern Annular Mode playing a corresponding role in the Southern Hemisphere. But why should the dominant EOF of a dynamical system appear to play a key role in determining the response to some imposed forcing?

One of Einstein's papers, published in his 'Annus Mirabilis' of 1905 was on the theory of Brownian motion (*Einstein, 1905*). In this paper, Einstein established that the same random forces which cause the erratic movement of a particle in Brownian motion would also cause drag if the particle were pulled through the fluid. This result in turn became developed and generalised to the so-called fluctuation-dissipation theorem in statistical thermodynamics, quantifying the relation between the fluctuations of a system in thermal equilibrium and the response of the system to applied perturbations. *Leith (1975)* has applied the fluctuation-dissipation theorem to understand the forced response of the atmosphere. Let

$$\begin{aligned} \dot{\mathbf{X}} &= \mathbf{F}[\mathbf{X}] \\ \dot{\mathbf{X}}' &= \mathbf{F}[\mathbf{X}'] + \delta\mathbf{f} \end{aligned} \tag{2}$$

and $\delta\mathbf{X} = \mathbf{X}' - \mathbf{X}$, then (Leith's version of) the fluctuation dissipation theorem states

$$\delta\overline{\mathbf{X}} = \mathbf{L}\delta\mathbf{f} \tag{3}$$

where the overbar represents a long time average and

$$\mathbf{L} = \int_0^\infty \mathbf{C}(\tau)\mathbf{C}^{-1}(0)d\tau \tag{4}$$

where $\mathbf{C}$ is the lag-τ covariance matrix of $\mathbf{X}$.

We see in the fluctuation-dissipation theorem a mathematical statement of the notion that the response of a system to some prescribed forcing is conditioned by that system's internal modes of variability, i.e. the response to the forcing will be conditioned by the projection of $\delta\mathbf{f}$ in the direction of the leading eigenvectors of $\mathbf{L}$.

Compounding this problem, operators associated with dynamical evolution of small perturbations are almost never self adjoint (*Palmer 1996*). Let us focus on a system linearised about a stationary basic state. By definition, an initial perturbation which projects entirely onto the system's leading mode of variability will evolve into a perturbation which also projects entirely onto that mode. However, such an evolved perturbation does not in general have optimal projection onto this mode. Rather, an initial perturbation which evolves into one with the largest projection onto the system's leading mode of variability, will itself project onto the adjoint of that mode (*Farrell and Ioannou, 1996*). In cases where modes are close to degenerate, the adjoint mode will be almost orthogonal to the mode itself (see Fig 4). Correspondingly, a forcing in an orthogonal direction can produce a substantial response in the direction of the mode itself. Figure 3d shows a case where the system's response is orthogonal to a prescribed forcing.

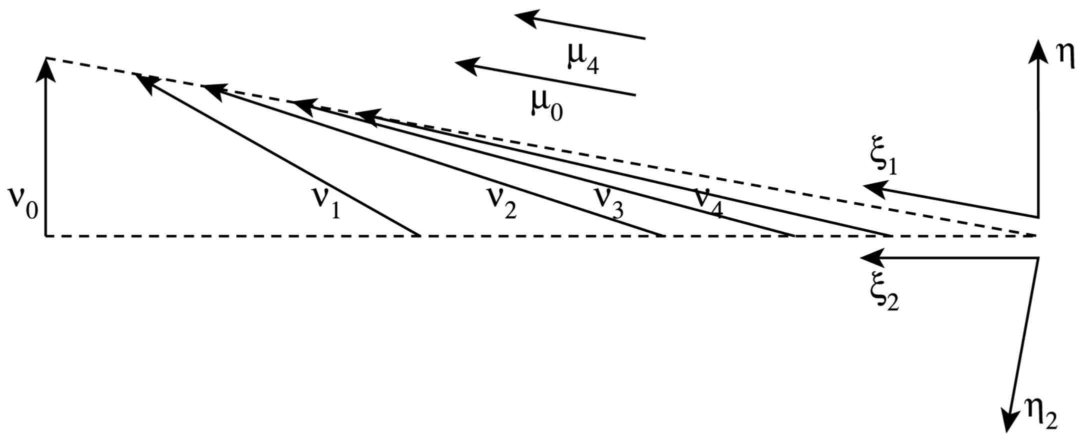

Figure 4: Schematic illustration of perturbation growth in a highly non-self adjoint system. The perturbation μ_0 projects onto the dominant eigenvector ξ_1 and decays. The optimal perturbation ν_0 projects onto the adjoint η_1 of ξ_1 and grows over a finite period. The non-self adjointness of the system is reflected in the near degeneracy of the first two eigenvectors ξ_1 and ξ_2

Putting these two effects together, we can conclude that it is on the one hand, the dependence of the forced response of a system to the system's internal modes of variability, and the non-self-adjointness of these modes on the other hand, that makes diagnosis of model bias so difficult.

What to do? The results above relate to problems of diagnosing the cause of model bias from integrations where the model has asymptoted towards its climatology. In principle, this suggests a relatively simple solution: perform the diagnosis well before the model integrations have asymptoted to climatology. We study this in the section below.

4. Can a 6-hour weather forecast help determine Earth's climate 100 years from now?

Climate change is the defining issue of our age, yet predictions of climate for the end of this century remain remarkably uncertain. In large part this is because the amplification of increases in greenhouse gases by cloud-radiative interactions remains uncertain. This is turn arises because the parametrised representation of clouds themselves is especially uncertain.

The issue of uncertainty was put into sharp focus by analysis of the climateprediction.net ensemble of climate change projections. According to *Stainforth et al (2005)*, climate sensitivity is predicted to be as large as 12K or more. As discussed in *Rodwell and Palmer (2007)*, many of the models producing such strong global warming signals had convective parametrisations with anomalously small values of the convective entrainment parameter.

Although the amplification of the effect of enhanced CO_2 by convective cloud systems will occur on timescales of decades, the intrinsic timescale associated with a deep convective system itself is typically on the order of hours. Hence it should in principle be possible to assess whether the anomalously small values of convective entrainment are realistic or not, by studying the performance of such models in short-range weather prediction mode.

This can indeed be done, as reported in *Rodwell and Palmer (2007)*. The technique is illustrated in Fig 5 based on the technique proposed by *Klinker and Sardeshmukh (1992)*. Essentially the idea is to look at the mean ‘analysis increment’ averaged over a month of four-times-a-day atmospheric analyses. Here an analysis increment is defined as the difference between a 6-hour forecast and the corresponding objective analysis of the contemporary observations, valid at the same time as the forecast. These objective analyses are used to initialise weather predictions. Over a sufficiently long time series of analyses, the mean analysis increment should be close to zero. However, if the model is biased against the observations, then the mean analysis increment will be non-zero.

This approach to model diagnosis overcomes the constraints of the fluctuation-dissipation theorem because the approach is based on a diagnosis of model output well before any asymptotic climatological state is reached. In practical terms this means, for example, that mean analysis increments associated with an error in the representation of orographic gravity wave momentum-flux convergence will be largest in the momentum equation and in the lower stratosphere above regions of large sub-grid orography where such waves tend to break. Indeed this was one of the first applications of this technique (*Klinker and Sardeshmukh, 1992*).

Rodwell and Palmer (2007) found that the mean analysis increment in a model with anomalously small entrainment parameter (as used in the climateprediction.net experiments) was substantially larger than that from a model with more typical values for this parameter.

Analysis increments provide a diagnostic tool that is used to assess routinely biases in the ECMWF system (*Rodwell and Jung, 2008)* We propose here that it could prove an invaluable tool for climate modelling, in reducing bias in models, and in reducing uncertainty in projections of climate change.

However, in order to implement such a tool, the modelling system must have data assimilation capability. Currently, climate prediction models do not typically have this capability. However, in recent years, the concept of seamless prediction (*Palmer et al., 2008*) is bringing weather forecast and climate prediction models closer together. This will allow this technique to be explored more thoroughly in climate prediction mode.

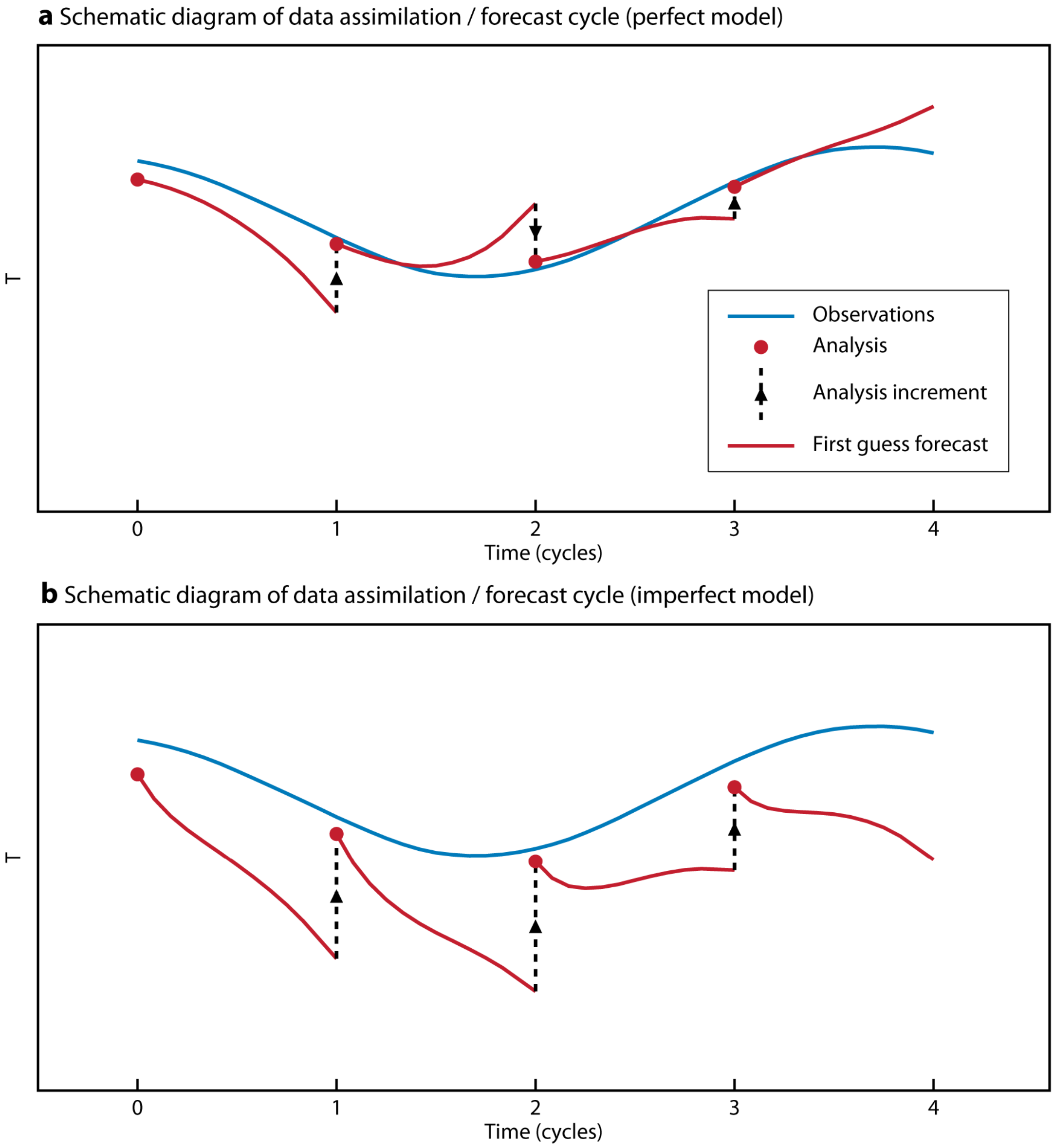

Figure 5a) Schematic diagram of data assimilation for a perfect model. b) as a) but for an imperfect model.

5. Bias of the second kind

Have we solved the problem of diagnosing the causes of model bias? That is, is it sufficient to focus on the initial tendencies of the model from an ensemble of initial states? Clearly this technique will not work for diagnosing very long-timescale processes, e.g. associated errors in the representation of the carbon cycle. On the other hand, many of the important uncertainties in climate models are associated with rather fast timescale processes linked to clouds, boundary layer turbulence etc.

However, as discussed in this section, there is a second type of model error which will not be revealed by this type of diagnosis.

To see this, let us return to the *Lorenz (1963)* system, this time written in terms of the three principal components of the model (*Selten 1995*):

$$\begin{aligned}\dot{a}_1 &= 2.3a_1 - 6.2a_3 - 0.49a_1a_2 - 0.57a_2a_3\\ \dot{a}_2 &= -62 - 2.7a_2 + 0.49a_1^2 - 0.49a_3^2 + 0.14a_1a_3\\ \dot{a}_3 &= -0.63a_1 - 13a_3 + 0.43a_1a_2 + 0.49a_2a_3\end{aligned} \tag{5}$$

Now, it turns out that the third principal component only explains about 4% of the variance of the total system. We might therefore consider parametrising the equation for the third principal component in the following form

$$\begin{aligned}\dot{a}_1 &= 2.3a_1 - 6.2a_3 - 0.49a_1a_2 - 0.57a_2a_3\\ \dot{a}_2 &= -62 - 2.7a_2 + 0.49a_1^2 - 0.49a_3^2 + 0.14a_1a_3\\ a_3 &= P(a_1, a_2; \alpha, \beta ...)\end{aligned} \tag{6}$$

where P is some deterministic formula and $\alpha, \beta ...$ are parameters. Figure 6 shows integrations of the full model (equation(5)) and the parametrised model (equation(6)) where $P = \alpha a_1 + \beta a_2$.

It can be seen that the long-term climate of the parametrised model is clearly not chaotic. This is a consequence of the Poincaré-Bendixon theorem, whereby the state space of a chaotic system based on autonomous differential equations must have at least 3 dimensions ((6) is an autonomous system with only two degrees of freedom). But on the other hand, it can also be seen that the parametrised model is quite accurate for short-range forecasts. In this case, the analysis increment approach would not diagnose the fault with the parametrised model, since in the short range the parametrised model is clearly skilful.

What is this fault? Here the model deficiency lies in the use of a deterministic function P for the parametrisation: the specific linear form above is irrelevant. If we replace the deterministic parametrisation with a stochastic parametrisation

$$\begin{aligned}\dot{a}_1 &= 2.3a_1 - 6.2a_3 - 0.49a_1a_2 - 0.57a_2a_3\\ \dot{a}_2 &= -62 - 2.7a_2 + 0.49a_1^2 - 0.49a_3^2 + 0.14a_1a_3\\ a_3 &= \beta\end{aligned} \tag{7}$$

where β is a stochastic variable, then the broad structure of the Lorenz attractor is simulated reasonably well, as shown in Fig 7a. The amplitude and temporal autocorrelation has to be correctly tuned to give this structure. Figure 7b shows a simulation of equation (7) with weaker noise. The simulation shows a bias in both mean state and internal variability.

Hence, we see that there is more to parametrisation error than the determination of the parameters or the functional form of the parametrisation. The very assumption of a deterministic link between the unresolved scales and the resolved scales is here brought into question. We will refer to a bias resulting from such a structural uncertainty as a 'bias of the second kind'. The development of stochastic parametrisations in numerical weather prediction models addresses some of these deficiencies (*Palmer, 2001*; *Palmer and Williams, 2009*). In a nonlinear system, deficiencies in the lack of sub-grid variability can lead to systematic bias in the climate model, but, as suggested in Fig 6, the timescale for the development of such biases can be quite slow. Hence, how are we to diagnose model deficiencies associated with the lack of sub-grid variability?

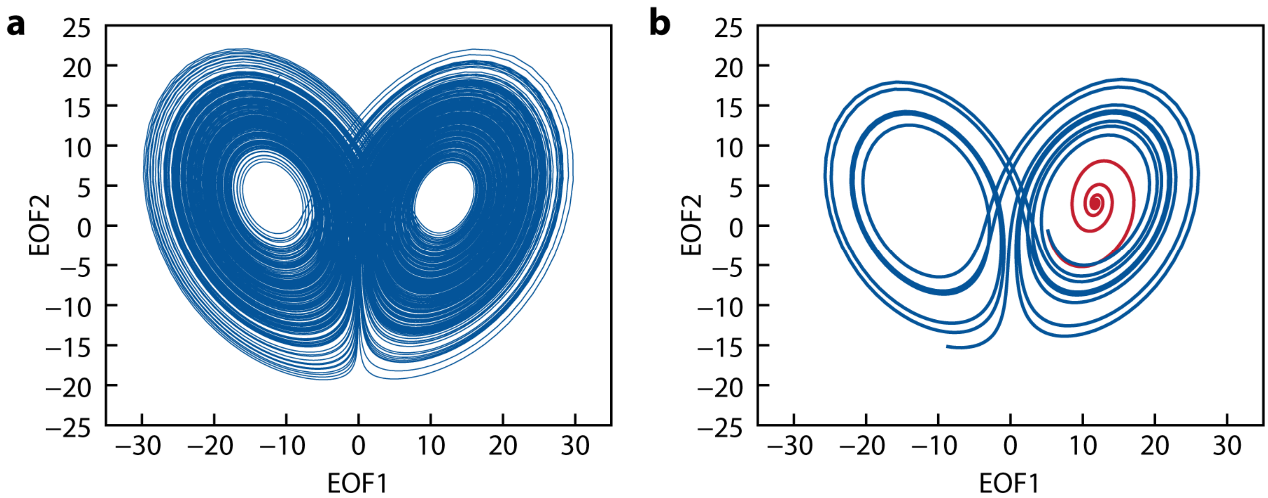

Figure 6a) Trajectories of Lorenz model in a principal component basis. b) blue lines as a), red lines for a 2D model with deterministically-parametrised third principal component.

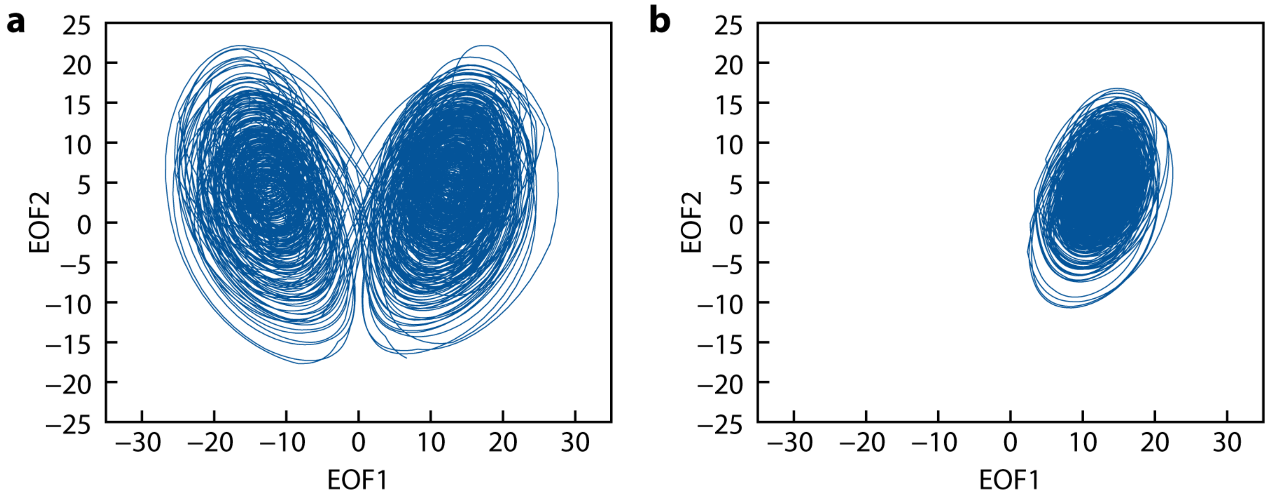

Figure 7a) Lorenz model with stochastically-parametrised third principal component. b) as a) but with smaller-amplitude noise.

One idea that seems to have considerable potential is by developing stochastic parametrisation through coarse-grain budgets of cloud-resolving models (*Shutts and Palmer, 2007).* For example, the cloud resolving model may have a resolution of 1 km and one estimates coarse-grain budgets over boxes of size c. 100 km, a typical dimension for a climate model grid box. Here one treats cloud resolving model output as a surrogate for truth. By treating the cloud resolving model output as truth, one has exact estimates of the sub-100-km grid tendencies. Based on this one can estimate probability distributions of sub-100-km grid tendencies, conditioned on the 100 km average flow. This method has been used to provide a partial validation for stochastic parametrisations used at ECMWF: both the Stochastically Perturbed Parametrisation Tendency Scheme, and the Stochastic Backscatter Scheme (*Palmer et al 2009*).

6. Conclusions

Climate prediction models provide the scientific input which underpins climate change mitigation treaties and adaptation strategies. Whilst there has been considerable improvement in climate simulations over recent years, climate models have quantifiable shortcomings and develop biases of magnitude comparable with the climate change signals such models are trying to predict. It is clearly important to try to reduce these biases, and yet diagnosis of model error is difficult not least because of the problem of compensating errors: the response to errors in the representation of two quite different processes in a climate model can partially cancel each other out. This problem of compensating errors can be illustrated in relatively simple nonlinear models, but may ultimately be linked to rather generic properties of nonlinear systems.

A technique is proposed to overcome some of these problems, based on the concept of analysis increments. However, this technique requires the model to come with a data assimilation system: currently few climate models have this capability. However, with the development of seamless prediction systems, there is a prospect of significant advances in the future.

Finally, a 'bias of the second kind' has been discussed. This is not associated with the functional form of sub-grid parametrisations, nor of the values of the free parameters associated with such parametrisations, but rather with the fact that that such parametrisations are deterministic. The development of stochastic parametrisations, aided by coarse-grain budget analyses from cloud resolving models, may provide the means to reduce such biases.

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