Breeding and predictability in coupled Lorenz models

E. Kalnay, M. Peña, S.-C. Yang and M. Cai

Department of Meteorology
University of Maryland, College Park 20742 USA

Abstract

Bred vectors are the difference between two nonlinear model integrations, periodically rescaled to avoid nonlinear saturation of the instabilities of interest. We present applications of breeding vectors in a coupled version of the Lorenz (1963) model. The bred vector growth can be used to reliably predict which will be the last orbit in each of the two regimes of the Lorenz model, and how long will the next regime last. A new, coupled Lorenz model with a fast and slow component exhibits very irregular chaotic behavior in the fast solution. We show that by using the slow variables with appropriately large amplitudes and intervals in the rescaling of the bred vectors, it is possible to obtain the instabilities of the slow, coupled system even if the amplitude of the "fast noise" is as large as the "slow signal". Conversely, using fast variables with shorter rescaling periods isolates the fast instabilities.

1. Introduction

Forecast errors can originate from errors in the initial conditions that, due to the chaotic nature of the atmosphere grow with time, or from model deficiencies (which we do not consider here). Because the error growth is not uniform, but is associated with instabilities of the background flow, forecast errors tend to be dominated by relatively large "errors of the day" intermittent in space and in time. Bred vectors are the difference between two nonlinear model integrations, periodically rescaled. Kalnay and Toth (1994) conjectured that they represent the same instabilities that generate the "errors of the day" and can therefore be used to estimate the shape of the forecast errors.

We review the method to generate and properties of bred vectors (Section 2). In section 3 we show that breeding growth rates provide reliable forecast rules for regime transition in the Lorenz (1963) 3-variable model, and in section 4 test the conjecture of Toth and Kalnay (1993) that it is possible to create bred vectors associated with fast or slow modes in coupled systems using a coupled Lorenz (1963) model.

2. Breeding in coupled systems

Breeding (Toth and Kalnay, 1993, 1997, Kalnay, 2002) was developed as a method to generate initial perturbations for ensemble forecasting at the National Centers for Environmental Prediction (NCEP). The method involves simply running the nonlinear model used for the control a second time, periodically subtracting the control from the perturbed solution, and rescaling the difference so that it has the same size as the original perturbation. The rescaled difference (a bred vector) is added to the control run and the process repeated. In the context of data assimilation, the rescaled difference is added to the analysis (Fig. 1). Bred vectors are a nonlinear generalization of leading Lyapunov vectors. Their growth rate is a measure of the local instability of the flow.

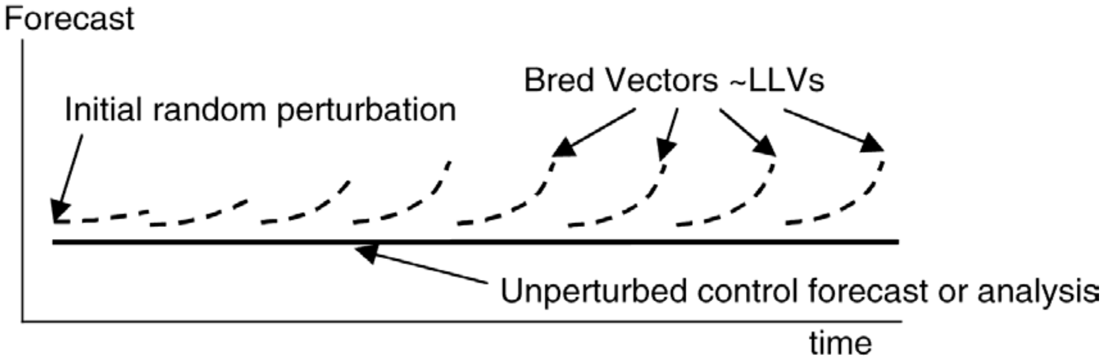

Figure 1 Schematic of the method to generate bred vectors

Toth and Kalnay (1993) noted that the use of nonlinear perturbations in breeding allows filtering unwanted fast, small amplitude, growing instabilities like convection. They pointed out that using rescaling amplitudes in the range of 1m to10m in the 500hPa height, the bred vectors in the operational global atmospheric model were independent of the amplitude and grew at a rate of about 1.5/day. With amplitudes of 1cm, however, the bred vectors had a completely different behavior: they were mostly tropical perturbations generated by convection that grew at a rate larger than 5/day (Toth and Kalnay, 1997). These irrelevant fast convective instabilities saturated out and were therefore not observed when larger amplitudes were used (Fig. 2a). With even smaller amplitudes, it is possible to observe growth rates of over 40/day in a tropical primitive equations model (Jim Geiger, pers. comm., 2001).

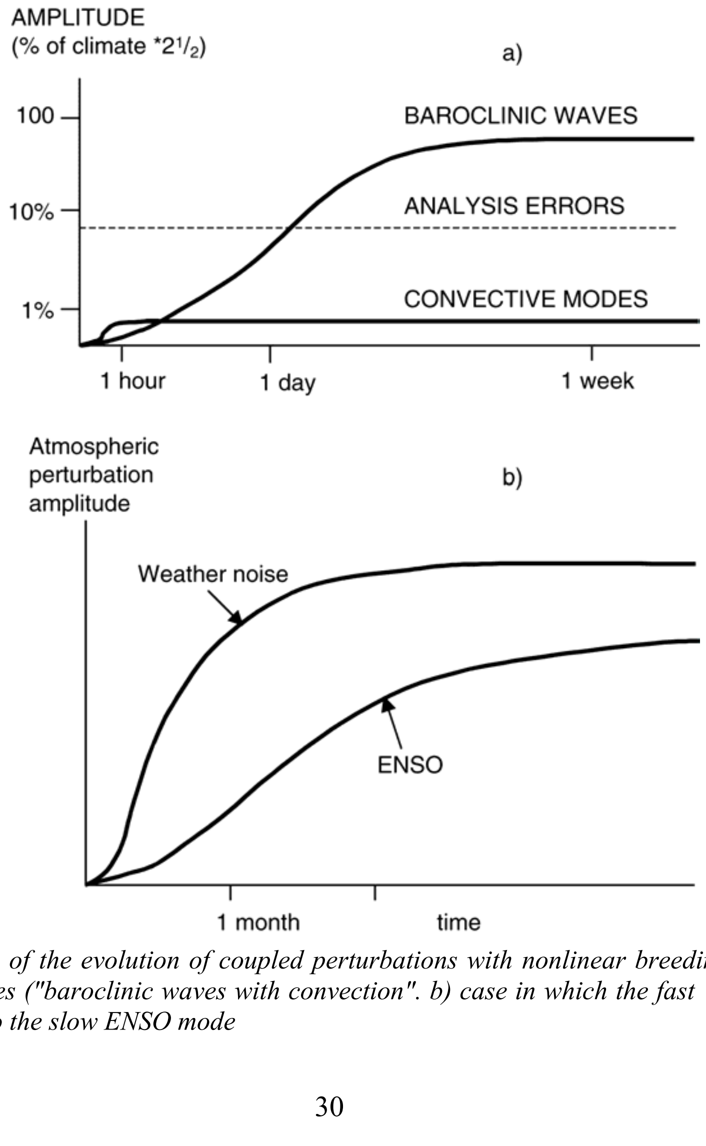

Figure 2 Schematic of the evolution of coupled perturbations with nonlinear breeding. a) case of small amplitude fast modes ("baroclinic waves with convection". b) case in which the fast "weather noise" has amplitude similar to the slow ENSO mode

Toth and Kalnay (1996) suggested that a similar idea might in principle be used to isolate the instabilities of the coupled ocean-atmosphere for ENSO predictions. This idea was tested by Cai et al (2002) with a simple Cane-Zebiak model, which has a diagnostic atmosphere. However, with a full coupled ocean-atmosphere GCM, the situation is more difficult than that described by the schematic figure 2a. For the atmosphere, the coupled slow ENSO signal is not larger than the "weather noise" (Fig. 2b). In this case is not clear a priori whether breeding with the slow ocean variable can be successful. In order to study this we performed experiments with a classic Lorenz (1963) model and with a coupled model with a fast and a slow component.

3. Results: breeding with a coupled slow/fast Lorenz model

3.1. Coupled model equations

We modified a simple coupled Lorenz equations system from a MATLAB program developed by Jim Hansen (pers. comm., 2002) by adding coupling on the z-variables, and by changing the spatial scaling parameter S and the offset parameter O from his version. The time-scale parameter is $\tau = 0.1$ for all experiments.

Fast equations

$$\frac{dx_1}{dt} = \sigma(y_1 - x_1) - C_1(Sx_2 + O)$$
$$\frac{dy_1}{dt} = rx_1 - y_1 - x_1 z_1 + C_1(Sy_2 + O)$$
$$\frac{dz_1}{dt} = x_1 y_1 - bz_1 + C_1^*(Sz_2)$$

Slow equations

$$\frac{1}{\tau}\frac{dx_2}{dt} = \sigma(y_2 - x_2) - C_2(x_1 + O)$$
$$\frac{1}{\tau}\frac{dy_2}{dt} = rx_2 - y_2 - Sx_2 z_2 + C_2(y_1 + O)$$
$$\frac{1}{\tau}\frac{dz_2}{dt} = Sx_2 y_2 - bz_2 + C_2^*(z_1)$$

3.2. Example 1: uncoupled classic Lorenz model (C_1=C_1^*= 0)

We first perform breeding on an uncoupled Lorenz (1963) model integrated with a Runge-Kutta scheme with time steps Δt=0.01, and a second run started from an initial perturbation $\delta \mathbf{x}_0$ added to the control at time t_0. Every 8 time steps we take the difference $\delta \mathbf{x}$ between the perturbed and the control run, rescale it to the initial amplitude and add it to the control. We also measure the growth rate of the perturbation per time step as $1/8 * \ln |\delta \mathbf{x}| / |\delta \mathbf{x}_0|$. It is clear (Fig. 3a) that with this simple procedure we can estimate the stability of the attractor. Moreover, the growth rate measured by breeding (Fig. 3b) provides remarkably robust and precise "forecasting rules" that could be used by a forecaster living in the Lorenz attractor to make "extended range forecasts" about when will the present regime end, and how long will the next regime last. The presence of a red star shows bred vector growth in the previous 8 steps was greater than 1.8, and can be used to forecast that the next orbit will be the last one in the current regime. Furthermore, the presence of only one or two red stars indicates that the next regime will be short, but sustained large growth shown by several red stars can be used to forecast that the next regime will be long lasting. The simple computation of bred vector growth, similar to the local Lyapunov growth rate, makes the Lorenz attractor quite predictable, and it is also a good indicator of future ensemble spread (not shown).

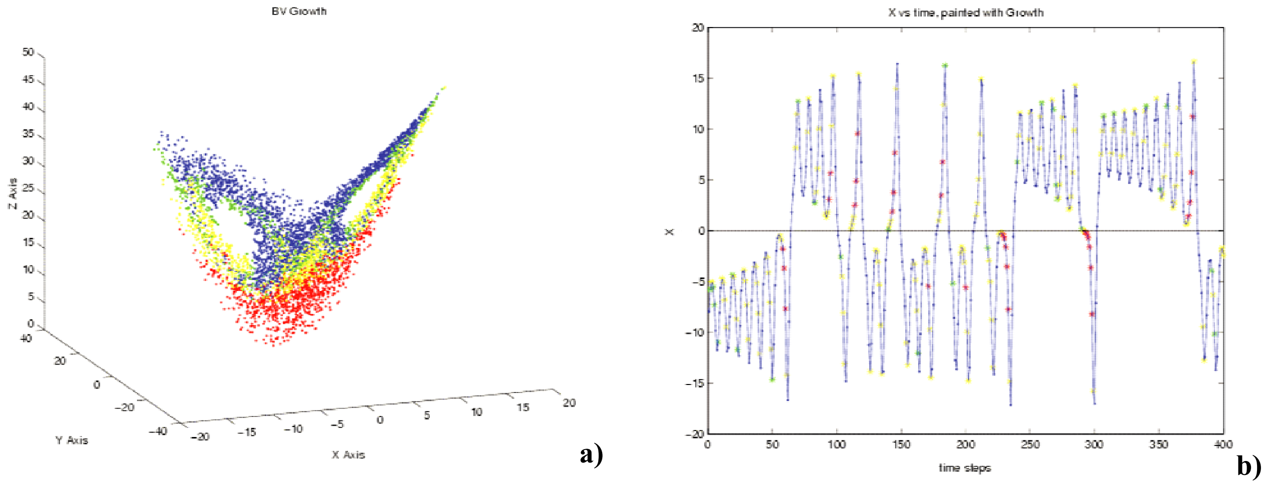

Figure 3 a: The Lorenz classic attractor colored with the bred vector growth. Red indicates a growth of more than 1.8 over 8 steps. Fig. 3b: X(t) for the classic Lorenz model with red stars providing "forecasting rules" (see text)

3.3. Example 2: "Baroclinic waves coupled with convection"

In this example (similar to the situation described in Fig. 2a) we chose to have the slow waves with an amplitude 10 times larger than the fast waves, and only weak partial coupling ($\tau = 0.1$, $S = 0.1$, $C_1 = C_2 = 0.15$, $C_1^* = C_2^* = 0$) in order to test whether it is possible to breed either the Lyapunov vectors corresponding to large amplitude, slow motion, or the small amplitude, fast Lyapunov vectors, by using either large or small amplitudes in the rescaling. Figure 4a presents the exponential growth rates for the slow modes when using the slow variables for breeding, and Figure 4b the exponential breeding growth rates computed using the fast modes and a more frequent rescaling frequency. It is clear that the choice of the amplitude and frequencies allows for a simple separation of the two types of growth rates corresponding to the two types of modes. This case corresponds to the situation that had already been tested by Toth and Kalnay (1993, 1997) and others who used breeding to isolate high amplitude mid-latitude waves with a global atmospheric model.

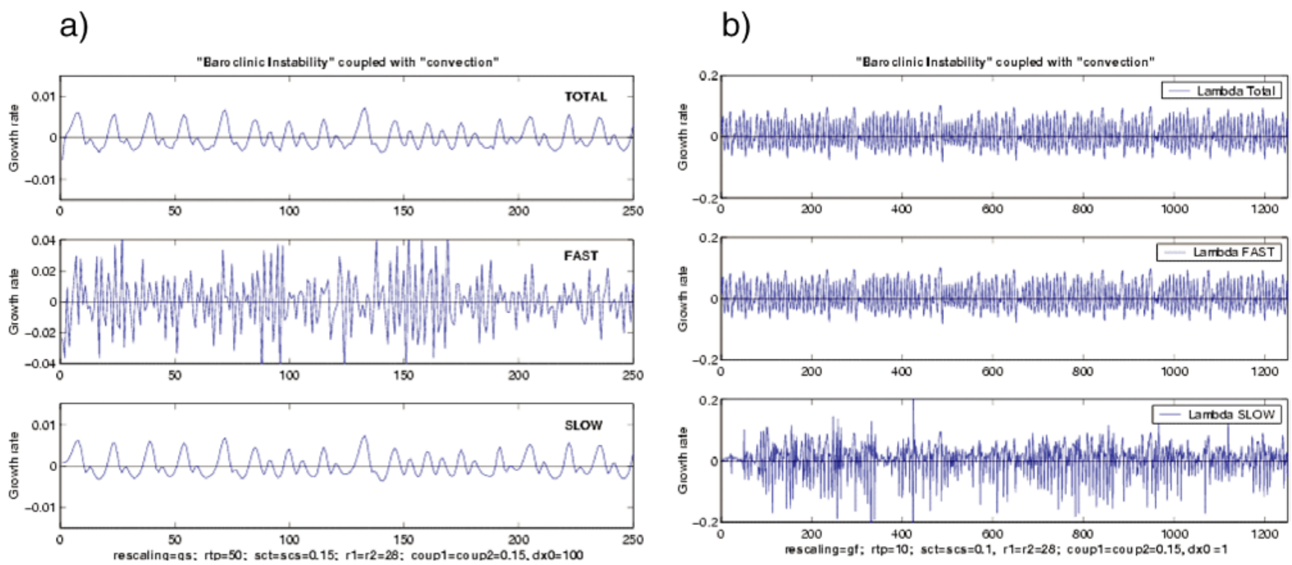

Figure 4 growth rates for the slow variables (bottom), fast variables (middle) and total growth rate (top) in the coupled Lorenz model with small amplitude fast component. a) rescaling with slow variables and long interval. b) rescaling with fast variables.

3.4. Example 3: "ENSO modes in a coupled ocean-atmosphere"

The second coupled case that we test, corresponding to Figure 2b, is considerably more challenging, since the fast and slow waves now have similar amplitudes. We also included stronger coupling ($\tau = 0.1; S = 0.5; C_1 = C_2 = 1.0 ; O = -11$). The resulting attractor is quite interesting, having lost the orderly "butterfly" appearance of the original Lorenz (1963) model. Figure 5a shows that the slow variables have one regime that occurs most of the time ("normal") and a less frequent regime ("El Niño"). At the same time, the atmosphere is much more chaotic than the typical Lorenz model ("weather noise"). We found, not surprisingly, that by frequent rescaling using the fast variable we recovered the large amplitude fast modes that are derived from linear Lyapunov theory. However we were also able to isolate the slow, coupled mode by rescaling with the slow variables, large perturbation amplitudes and a longer rescaling interval, just as in the case of small amplitude fast component. These results (not shown) were similar to those obtained in the previous case.

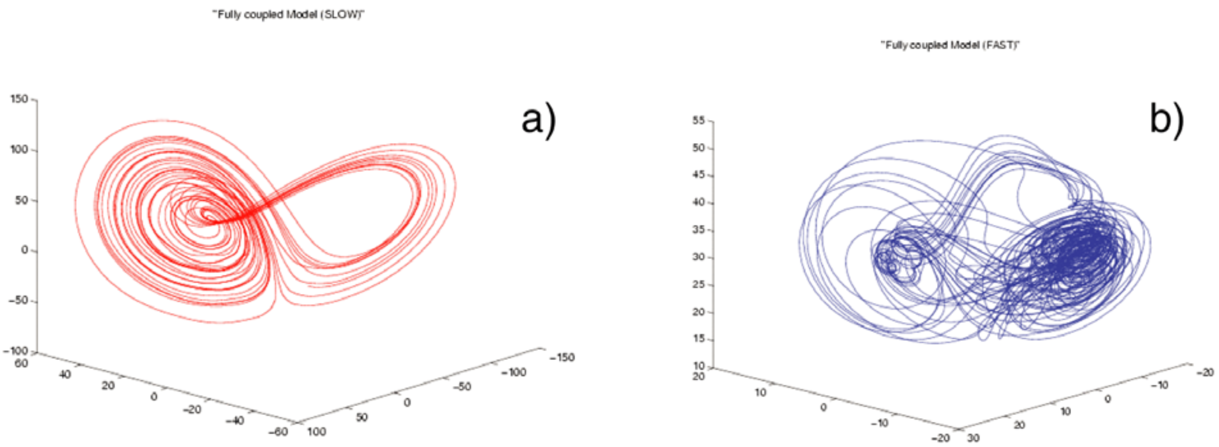

Figure 5 Fully coupled Lorenz model with large amplitude fast component ("ENSO"). a) slow variables, b) fast variables

4. Summary

We presented examples with the Lorenz models that indicate that with simple breeding, we can make accurate "long-range forecasts" of regime changes for the classic chaotic Lorenz 3-variables model. In addition, coupling a fast and a slow Lorenz model, we can do breeding of the slow modes, even when the fast and slow modes are of similar amplitude. This approach can be directly applied to the El Niño coupled instabilities (Cai et al, 2002). These results are important because, as pointed out in the lecture by A. Timmerman (this volume), the problem of coupled systems with different time scales is pervasive in geophysics, and standard Lyapunov methods (based on the use of the linear tangent model) are only able to isolate the fastest type of Lyapunov vectors (frequently irrelevant). Timmerman pointed out the existence of some methods to address this problem, such as replacing the fast component with a "diagnostic" response, but they require considerable additional development and insight. Singular vectors (unless appropriately modified) will also yield the fastest type of growth present in the system, since they are obtained using a linear tangent model and its adjoint. The approach to obtain either fast or slow bred vectors that we tested is computationally inexpensive, and does not require changes in the coupled model. Preliminary results with the NASA NSIPP coupled ocean-atmosphere GCM indicate that rescaling with slow ENSO variables (such as Niño-3 SST) can be used to isolate the slow coupled system instabilities.

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