INTRODUCTION

During the past ten years the use of semi-Lagrangian techniques in numerical weather prediction has matured rapidly, from demonstrations of their usefulness in shallow-water models and preliminary experiments in three-dimensional models to widespread operational implementation. The increase in resolution of the Centre's operational model to T213 and 31 levels in 1991 would certainly not have been possible without the efficiency gains provided by incorporating a semi-Lagrangian scheme. Besides overcoming the timestep restriction of Eulerian methods, the semi-Lagrangian technique provides an accurate and efficient multi-dimensional advection scheme to which shape-preserving properties can easily be added if required.

Interesting developments in semi-Lagrangian methods are continuing, and there is still much to be learned about their incorporation into complex numerical models of the atmosphere. It was therefore timely to hold a workshop on semi-Lagrangian methods at ECMWF in November 1995. The workshop followed the usual pattern of 1½ days of lectures, followed by discussions within working groups and a final general session to discuss the conclusions of the working groups.

These proceedings contain the report of the working groups, followed by the texts of the invited lectures. ECMWF thanks all the participants for their contributions to a successful workshop.

REPORT OF THE WORKING GROUPS

Two working groups were set up to discuss issues raised during the presentations at the Workshop. Since there was some overlap in the topics addressed by the two groups, their reports have been combined.

1. Linear grid

The linear grid uses (2M+1) points (M being the triangular spectral truncation) along the direction of the Fourier transforms and (2M+1)/2 Gaussian latitudes for the Legendre transforms. This grid allows exact transformation of a linear term in spectral space to grid-point space and back but does not eliminate aliasing in the computation of the quadratic terms of the equations using the transform method as the usual (quadratic) grid does. *Côté and Staniforth* (1988) were the first to demonstrate that such a grid could be used for a model with a semi-Lagrangian treatment of advection.

Several groups have tried the linear grid in their spectral models, and the Canadians are using this grid in their operational configuration. HIRLAM has tested it in the spectral (bi-Fourier) version and NCAR in their community climate model. Also Météo-France has tried it in the stretched configuration of ARPEGE, and ECMWF has performed some experimentation on this kind of grid.

So far, nobody has reported problems of stability in using this grid in conjunction with the semi-Lagrangian advection, and only Météo-France has reported poor preliminary results as compared with the usual Gaussian grid.

In view of this evidence, the working group concluded that "the linear Gaussian grid looks extremely promising and its implementation should be pursued vigorously by ECMWF".

The only aspect of the linear grid utilization needing more attention is that of "spectral blocking": due to the presence of aliasing in the computation of non-linear terms, some build-up of energy is apparent in the high-wavenumber end of the spectra at the beginning of the integrations, which seems to stop after a while. This energy can be removed by horizontal diffusion and therefore this issue is somewhat related to the issue of horizontal diffusion.

2. Two-time-level scheme

Two-time-level semi-Lagrangian schemes are potentially twice as efficient as their three-time-level counterparts. In the context of the scheme used at ECMWF, this gain appears to be already realised at T213 resolution, with a doubling of the standard timestep from 15 minutes to 30 minutes. At lower resolution (T106 and T63), there is however a degradation of the scores when the two-time-level scheme with a timestep of 60 minutes is compared against the standard three-time-level scheme with a timestep of 30 minutes.

The Centre is encouraged to complete the development of the two-time-level scheme with a view to operational implementation. Several avenues were suggested for studying the problem at lower resolution. In particular, concern was expressed over the choice of reference temperature in the splitting of linear and nonlinear terms, especially where this choice results in large nonlinear terms which are then extrapolated in time. Although it is claimed that the choice of a warm reference temperature removes the need for decentering, it is possible that the linear interpolation of right-hand-side terms performs a similar damping function. This could be checked both by extending the stability analysis of the two-time-level scheme to include the effects of interpolation and by model experimentation. Also, the stability of the advective treatment of Coriolis terms in a two-time-level scheme should be checked by analysis.

The time-extrapolation of nonlinear terms in the two-time-level scheme is a potential source of problems, and could be avoided by treating these terms implicitly in time. This approach is being taken in the next-generation UKMO unified model, in the variable-resolution global model being developed at RPN, and in the model developed by *Bates et al.* (1995) and its three-dimensional extension. These developments should be closely monitored, though the application in the framework of a spectral model is rather unattractive.

3. Conservation issues

A disadvantage of most semi-Lagrangian schemes today is that they do not formally conserve integral invariants such as total mass, water, chemical constituents, total energy, angular momentum, enstrophy, etc.

This may not be a problem in medium-range forecast applications. For long simulations in seasonal forecasting and in climate simulations, however, lack of conservation might have serious consequences. The total mass, in particular, has been found to drift significantly if no corrections are applied during longer integrations.

In such a case, errors in the prediction equation for surface pressure clearly accumulate. The mechanism(s) leading to these errors are not known and we can detect only the error in the global mean surface pressure field. It seems likely, however, that the accumulating large errors in the mean surface pressure are accompanied by errors which are locally even larger and which may be systematically correlated with the pressure pattern. If such a correlation exists the internal dynamics of the model may be affected significantly. We do not know if this is the case, but it seems possible, and if so then the lack of conservation is a symptom of some perhaps more serious systematic errors.

Conservation of total mass may be obtained by a "mass-fixer" which restores the mean surface pressure to its initial value after each time step. Such a mass-fixer was tested by *Moorthi et al.* (1995) who restored the mean surface pressure after each timestep by multiplying the surface pressure everywhere by the required factor. This mass-fixer is similar to that used by *Williamson and Olson* (1994), except that they allow for variations in the total mass of water vapour. In this form of mass-fixer the horizontal pressure gradient is not affected by the restoration and the effect on the internal dynamics is therefore minimized. When comparing seasonally averaged fields from seventeen-month integrations with and without mass restoration *Moorthi et al.* (1995) found no significant differences. Thus, with this type of mass-fixer the restoration does not seem to affect significantly the simulated climate.

The restoration each time step with the same factor everywhere is arbitrary and the geographical distribution of the corrections is most likely wrong. Recently *Gravel and Staniforth* (1993) have presented an alternative mass-fix procedure where the restoration of the mean pressure is made only at selected points. They argue that the interpolation is likely to introduce errors that cause lack of conservation in areas of strong gradients, and they make corrections at exactly such points. A great deal of arbitrariness is, however, still present in their procedure with regard firstly to the magnitude of correction at each point and secondly by choosing not to do any mass restoring corrections at those points where the preliminary correction to fulfil monotonicity goes in the "wrong" direction.

Support for the hypothesis that the lack of conservation is likely to arise in areas of strong gradient comes from experiments at ECMWF with the "Eulerian averaged treatment of mountains" proposed by *Ritchie and Tanguay* (1996) in which the advected quantity in the continuity equation is much smoother than in the standard "semi-Lagrangian treatment of mountains". In these experiments the conservation of mass is much better than in the standard treatment. In any case, the experience at ECMWF is that the semi-Lagrangian scheme at T213L31 resolution conserves mass better than the Eulerian T106L19.

Mass is not the only important conserved variable. There are strong indications of problems with semi-Lagrangian advection of chemical constituents. The usual fixes introduce spurious transport of tracers, especially in the vertical. In the "cell-integrated method" these problems of non-conservation seem to be eliminated; however it needs to be further developed and tested. The cascade method of interpolation (*Leslie and Purser,* 1995) is an alternative method for exactly conserving mass and tracers.

This issue is not considered to be an immediate priority for ECMWF, but it may become more important when ozone is included in the model and data assimilation system.

4. Interface with physical parameterizations

The consistency of the coupling between the semi-Lagrangian advection and the physical parametrizations is an important issue. In the ECMWF model, the coupling is not done consistently. The physical tendencies are computed at the arrival point but using $t-\Delta t$ values. However, even though different groups use slightly different ways of coupling the semi-Lagrangian scheme with the physics, there seems to be no clear indication, at ECMWF or elsewhere, that this represents a significant problem.

One may ask if there is a way of computing a truly semi-Lagrangian physics (i.e., one in which the time tendencies are not computed as instantaneous tendencies but rather integrated over the time-step used by the dynamics). No clear answer to this question seems to exist. Nevertheless, it is felt that, for example, any source of negative humidity coming from the physics implies the presence of time truncation errors from treating the instantaneous tendencies as constant during the whole time step. Some groups have been using the physical tendencies averaged along the semi-Lagrangian trajectories; there is little evidence that this procedure provides any great advantage over the "Eulerian" approach. R. Bates reports marginal differences in scores between the two approaches. It is clear, however, that at higher resolution this might become a more important issue.

A related issue is the timestep-dependent behaviour of some physical parametrization schemes, and the possibility that this dependence contributes to different systematic errors between Eulerian and semi-Lagrangian versions of models.

Conclusion: although no specific solutions or strategies are proposed, most researchers feel the need for better ways of coupling the semi-Lagrangian schemes with the physical parametrizations.

5. Vertical boundary conditions for trajectories

In most current models, if the computed departure point falls outside the model domain (e.g., above the highest model level or below the lowest model level), it is artificially "pushed back" into the domain. This is probably a fictitious source or sink that may contribute to a lack of conservation, but the seriousness of this problem is not clear.

There are no signs that this is an urgent problem, and it is not considered to be a priority area for ECMWF to investigate. Other groups plan to investigate it in the near future in column models and by intercomparison studies with more sophisticated treatments which do not suffer from this problem.

6. Horizontal diffusion

The amount of horizontal diffusion used at T213L31 and other resolutions should be reassessed now that other issues have been resolved.

Experience elsewhere appears inconclusive. Bates et al. report that they are able to run their semi-Lagrangian scheme with no diffusion and no noise problems; a similar result is reported from UKMO. On the other hand, NCAR and others report that diffusion is required to obtain acceptable KE spectra.

Semi-Lagrangian schemes are inherently diffusive, so using such a scheme in place of a non-diffusive scheme could suggest that less explicit diffusion is required. However we believe that some horizontal diffusion is required to avoid an accumulation of energy at the smallest scales.

In the semi-Lagrangian ECMWF model, there is now no horizontal diffusion of humidity which is kept in grid-point space. However, rainfall near orography should be examined closely to see whether the horizontal diffusion can be tuned more effectively or even applied locally to improve the behaviour near orography. Possible inconsistencies in the application of horizontal diffusion (e.g., on non-horizontal surfaces, and for T_v but not q) should be addressed.

7. Interpolation issues

Experience has been varied on the optimal choice of schemes in the vertical with different groups using non-interpolating vertical schemes, or interpolating schemes either with or without monotonicity constraints.

Shape-preserving or monotone vertical interpolation has been identified as beneficial in removing vertical grid-point noise associated with stratospheric inertial instability; however, at ECMWF a large error has been observed in the treatment of the stratospheric tropical easterly jet if the quasi-monotone option is used in the vertical for the wind field.

Semi-Lagrangian vertical advection schemes have also been found to be less satisfactory than Eulerian at low vertical resolutions (fewer than 20 levels). Low resolution at the tropopause affects the semi-Lagrangian scheme more than the Eulerian.

There was a suggestion that pressure should be used as the vertical coordinate in the search for the departure points and for vertical interpolations. Nevertheless, the experience at both ECMWF and NCAR during the development of their respective semi-Lagrangian schemes was that the use of σ or η in the vertical part of the scheme makes no significant difference when compared with the use of either pressure or height. The issue may be revisited in the future, for example in the context of very high resolution modelling.

The behaviour of the semi-Lagrangian vertical advection schemes under different circumstances warrants further investigation.

In the horizontal, there is consensus that monotonicity preserving schemes have been advantageous for most model fields.

It is recommended that the Centre look at the use of cascade interpolation (*Purser and Leslie*, 1994) in the semi-Lagrangian scheme. This approach offers interesting computational economies, conservation properties and the possibility of higher-order spatial interpolation. Moreover, it may facilitate the development of the adjoint of the semi-Lagrangian scheme, both by simplifying its formulation and (when combined with higher-order interpolation) by improving its differentiability. However, some modification of the cascade algorithm will be required in order to make it work with the "reduced" Gaussian grid.

8. Very high resolution / nonhydrostatic models

It is an open question whether semi-Lagrangian schemes, with timesteps such that the Courant number is significantly greater than 1, will be useful for very high resolution models in which non-hydrostatic effects start to become important. The nonhydrostatic semi-Lagrangian MC2 model, for example, has so far not completely addressed the problem of orographic resonance (*Héreil and Laprise*, 1996). In tackling this problem, it will be necessary (but difficult) to distinguish between realistic and spurious orographic effects.

The usefulness of semi-Lagrangian schemes will probably depend on the phenomenon being studied - for example it may be possible to simulate accurately the steady-state response to orographic forcing, while the details of the transient response are wrong. At the same time, the ease with which monotonicity constraints can be applied in a semi-Lagrangian scheme may well be an important advantage. These considerations are worth bearing in mind; they are not of short-term concern for ECMWF, but in the longer term they will become important.

9. Orographic resonance

Recent developments have significantly reduced the difficulties related to spurious orographic resonance in semi-Lagrangian schemes. In particular, several groups (including ECMWF) have implemented the suggestion of *Ritchie and Tanguay* (1996) and found it beneficial. However, these developments do not represent complete solutions; the problem should continue to receive attention, especially with higher-resolution models in mind.

10. Higher-order time schemes

The results of *Leslie and Purser* (1995) suggest that higher-order time-integration schemes are worth considering even at current spatial resolutions, and there is an argument to suggest that they may be needed to maintain the advantage of semi-Lagrangian over Eulerian schemes at higher resolution. The results already demonstrated are for *forward* semi-Lagrangian trajectories. Two questions merit further study: can similar techniques be applied to backward trajectories as currently used in the ECMWF model, and do forward trajectories have significant advantages over backward trajectories?

11. Spectral vs gridpoint (finite-difference or finite-element) formulations

With the advent of the semi-Lagrangian method, many of the factors that previously gave an inbuilt advantage to the spectral formulation have been reduced in importance or eliminated. At the same time, gridpoint models which avoid the pole problem and provide a quasi-regular resolution over the sphere through the use of icosahedral or "cubic" grids are enjoying something of a revival.

Meanwhile, a significant drawback of the spectral formulation, the Gibbs phenomena associated with the representation of sharp gradients in the fields, is ameliorated with the linear grid or totally avoided as in the humidity field, by keeping it in gridpoint space. In particular the "ripples" in the spectrally fitted orography field are reduced by using a linear grid, provided that the underlying resolution of the field (before spectral fitting) remains the same.

An important problem which remains to be more carefully looked into is the influence of the orographic ripples and their interaction with the moisture variables (humidity, cloud, precipitation).

It is possible to argue that the spectral method is now used mainly for the ease of solving the Helmholtz equation of the semi-implicit method. Nevertheless, given the amount of work invested at ECMWF on the spectral transforms and its portability to MPP platforms, it was generally felt that the spectral method is reasonably efficient and there is not an urgent need for ECMWF to consider a different alternative. A close watch should however be kept on current developments in gridpoint methods.

It will be important for ECMWF to participate in dynamical core intercomparison experiments (e.g., *Held and Suarez,* 1994).

12. Summary of the main recommendations

The strongest recommendations of the Working Groups were the following:

* The linear Gaussian grid looks extremely promising and its implementation should be pursued vigorously by ECMWF.

* The Centre is encouraged to complete the development of the two-time-level scheme with a view to operational implementation.

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